A RISK CONTRIBUTION APPROACH TO ASSET ALLOCATION

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Abstract

The aim of this work[1] *is to verify whether efficient portfolios, obtained using traditional tools of asset allocation, provide real diversification in terms of risk, in addition to the division of capital into different asset classes. We will show how portfolios that seem diversified in their capital allocation are too heavily concentrated on risk allocation. To solve this problem we propose using a risk budgeting approach based on weighted equal marginal contribution to total risk. The diversification of risk will be effective in reducing the intensity and the length of drawdowns and in diversifying their source, with equal volatility to a traditional portfolio.*

1. Introduction

In Markowitz's theory the investor optimizes his portfolio according to a mean-variance approach. The weight of assets in a portfolio is based on their expected return, on their standard deviation and on their coefficient of correlation with the other assets included in the portfolio. Investors act following a procedure characterized by different steps. First of all they determine their preferred strategic asset allocation and then they select the individual assets to place in typical asset classes. If the investor believes that the market selected is efficient, he can add index funds or other products to the portfolio in response to market behaviour with low fees; otherwise he can adopt active strategies. Thus investors need to be skilled at assessing the best sectors of activity to include in a portfolio.

In this context, managers believe that the use of the mean-variance approach in building portfolios on the efficient frontier is an exercise in error maximization, since the assumptions used (i.e.

[1] This work is the result of a common effort by both authors. However, the first part of the paper (1, 2, 3) can be attributed to Claudio Boido and the second part to Giovanni Fulci.

expected returns, risk measures, and correlations) are many and the results obtained from decision variables (the weights of portfolios) are less meaningful as they depend on minor variations in inputs. Various mitigation techniques are suggested to solve the problem of these assessment errors. The full list, which is beyond the aim of this working paper, includes: techniques of estimation of the variance-covariance matrix in different market scenarios[2], skewness, kurtosis and non-normality[3], the Bayesian approach, resampling techniques[4] and robust optimization[5].

Apart from the solutions cited, there are other alternative approaches to beta portfolio construction, which are simpler in terms of the number of inputs compared to the mean variance approach and thus have less estimation errors. Attention has recently focused on the 1/n approach, assigning equal weight to all the assets in a portfolio[6] by naïve diversification.

Other researchers have proposedan approach called risk scaling[7], where every asset in the portfolio has a risk level, so each asset contributes to a fixed amount of the total risk. Intuitively, the risk weighting approach assigns more weight to less volatile assets. Since they are more stable over time and the assessment of their weight is more effective than the other techniques available for expected returns, by risk weighting we can obtain the right weight in portfolios which are stable in time and less exposed to assessment errors. In fact we don't need to assess the expected returns, but we are still dependent on errors in variance and covariance assessment.

This approach has the great benefit of ensuring much more stable diversification of a downside portfolio compared to the traditional approach of portfolio construction. This working paper aims to discuss these benefits.

So far the criticality of scores obtained with efficient frontiers, due to risk concentration in asset allocation[8], (Quian 2006, 2005) has not been clarified. In many cases the portfolios examined seem to be diversified by the distribution of weight over the various asset classes, but this situation is only apparent because risk is concentrated in only a few asset classes.

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[2] Regarding as covariance estimation in normal and turbulent markets, see Chow et al. For a review of techniques of estimation (shrinkage estimators and portfolio estimators) of the covariance matrix see Benninga et al.

[3] See Schreer (2004)

[4]See Michaud (1998)

[5] For a review of the techniques and applications of the approach based on robust optimization, see Fabozzi et al. (2007). We define robust optimization as the techniques for incorporating information concerning the uncertainty set for the parameters (expected returns and covariance) of the optimization model.

[6] De Miguel et al. carried out research using 14 different models of optimum portfolio construction. They verify that no portfolio, out of sample, is consistently superior to the "1/n" naïve approach, on the basis of the following evaluation criteria: out of sample Sharpe ratio, certain equivalent, and turnover. The models, whose aim is to reduce mistakes, obtain mediocre results because the effect of estimation errors on portfolio weights is very large. In this way the gain, in terms of optimum portfolio diversification, is compensated by errors in parameter estimation. Treynor (2005) verifies how the equal weights approach gives a better allocation to undervalued equities compared to overvalued ones and therefore better performance in relation to market indexes, thanks to mean reversion. See Neukirch (2008).

[7] See Clarke Roger G. (2006)

[8] See Quian (2005 and 2006). These papers both focus on a single portfolio composed of 60% equities and 40% bonds and the author does not verify the dynamic of risk concentration in portfolios of the efficient frontier, which is the aim of our working paper. We will discuss the effect on risk and portfolio drawdown later.

This risk concentration may be obvious along the edge of the efficient frontier (for example a 100% equity allocation concentrated in the risky part of the efficient frontier) but it is less obvious in the central part of the frontier. In this paper we propose to achieve more appropriate and stable risk diversification along all points of the efficient frontier. In this way managers will not be obliged to take position on 100% equities to reach a high risk/return.

The traditional approach to ensure high returns pushes managers to build portfolios dominated by more risky asset classes (i.e. equity). The final result is a portfolio dominated by only beta equity, without investing in other possible markets. Recently Markowitz (2007) noticed this problem concerning mangers who don't use leverage.

Markowitz's analysis maintains that managers who use leverage obtain a reduction of risk premium from risky assets. In fact managers that use long only strategies to compete with investors who use leverage buy risky assets on risk/return schemes that compress the risk premium.

This concentration on risky assets has brought about a migration towards portfolios characterized by 100% equities, due to the bigger expected risk premium in this class of asset.

In this paper we propose to use leverage with the aim of obtaining a high level of risk/return without compressing the risk premium. The final result is a less risky portfolio with equal expected return to those obtained on the efficient frontier.

Another proposal of this paper is to reduce (in-sample) maxdrawdown[9] with equal volatility in relation to efficient frontier portfolios. This mitigation of drawdown is a direct effect of the best downside diversification.

2. From Markowitz to active management

An active manger can compare their results against a benchmark, which immediately gives the value of an asset class. The skill of an active manager is to beat the benchmark because he is able to find the alpha component (that is the specific risk) in the market.

The most commonly used performance measures are: alpha, tracking error volatility, and information ratio. Specifically, the first measure is equal to extra-return in relation to the benchmark, tracking error volatility is the standard deviation of extra-return and the information ratio is the ratio between alpha and the tracking error volatility. Opinions on portfolio selection are given based on the expected alpha and tracking error, assessed according to the skill of the manager to value asset volatility and available news.

In conclusion, the manager tries to achieve the following objectives:

- High risk premium in medium term

[9] We specify that drawdown is the maximum loss of value for the investor, in percentages and for a prefixed period.

- Extra-return from active management
- To build a balanced portfolio as regards risk/return

In seeking to meet these goals the manager will have to diversify between the sources of risk premium (interest risk and credit risk) and active return. To obtain this result he tries to improve the value of the information ratio by increasing the investment opportunity and reducing the single constraints (which hinder active management). In this way we can balance the two different risks: systematic risk (beta) and active risk (alpha). The manager will have to focus his attention on risk budgeting, to avoid concentrating on equity with an inappropriate risk profile. This technique tries to optimize the portfolios in terms of expected alpha and expected tracking error volatility, with the aim of building portfolios characterized by specific returns for given target risks. The investor defines a maximum target tracking error volatility and he proceeds to allocate the tracking error, as a risk, to every asset class. In this view, risk is not eliminated but it is managed in order to obtain the expected return.

3. Risk budgeting in portfolio construction

Managers are concerned with monitoring risks in order to manage them efficiently, with the end goal of obtaining high returns for their clients. To clarify this problem, we shall examine the determinants of risk and return.

Investors divide portfolio return into three different parts:

- Risk free rate
- Risk premium (obtained from asset class returns)
- Active return

  In order to reach the expected return, the manager should be able to control risk by risk diversification. Risk is divided into systematic risk (beta) and active risk (alpha).

  Beta derives from passive management, that is the manager selects asset classes linked to index markets. This technique doesn't require particular skill and fees are very low. Systematic risk constitutes the most significant contribution to total risk. The selection of asset classes follows these rules:

  - Bond selection: bonds are chosen on the basis of specific features such as maturity, duration, credit ratings, government or corporate, foreign or domestic.
  - Stock selection: stocks are chosen according to the following features: small or large cap, sector, value or growth.

Ex-post assessment allows the manager to verify different weights in relation to a benchmark portfolio.

Active managers, who want extra-return from different asset classes, seek alpha (active) risk and in this case the skill of the manger is very important to the final result. Active risk is a zero sum game: while all passive managers make a profit if the stock market increases, some active managers could beat the market and others could choose a wrong position. The final result will be that some managers will win and others will lose. While active risk makes a small contribution to total risk, its cost is high because the result depends on the manager's skill, which can be measured by the information ratio[10].

In seeking alpha, active managers choose market sections not correlated with the market index [11]. If the manager observes different markets, he could select asset classes using the same ratio indicators in a different way, that is his selection could depend on different economic cycles. Some research[12] in the literature has investigated the the cross-sector factors that influence final payoffs, which would induce active managers to attribute greater weight to individual factors.

Total risk portfolios are therefore composed of systematic risk and active risk. According to one well-known paper[13] systematic risk weighs 90% of the total risk in relation to active risk. The following research shows that the weight of equity is much greater than the weight of bonds, so the usual distribution of 60% equity and 40% bonds doesn't permit effective diversification. Other authors[14] have recently affirmed that equity risk weighs 95% of the total risk in relation to the 5% weight of bond risk. The result obtained is justified by the non-linear volatility function and by the different volatility of stocks and bonds. In fact, stock volatility is 15%, compared to bond volatility, which is 4%. We can demonstrate that the total risk would be high if we put in a portfolio a stock component only slightly above 50% of the total risk. If we wanted to isolate active risk in a portfolio, the stock contribution would be greater (98%) than the contribution of bonds. This is because active risk is higher in equities than in bonds. Most active risk linked to equity originates from security selection (85%) and the residual components are financial products linked to market indexes.

This situation shows that it is incorrect to confuse capital allocation with risk allocation: for example if you share capital between stocks and bonds, you could not affirm that the percentage chosen for capital allocation has the same weight for risk allocation.

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[10] This is calculated from the ratio between tracking error and tracking error volatility. We know that a manager is skilled if the result is 0.2-0.5.

[11] Otherwise active returns are sought on equities with interesting levels of dividend yield and/or price earnings, with the aim of capturing the difference in relation to the equilibrium values.

[12] See Fame E., French K (1992); Lahonishok J., Shleifer A., Vishny R., (1994); Haugen R., Baker N. (1996).

[13]Brinson G.P., Singer B.D., Beebower G.L. (1991).

[14]Clarke R.G. (2006).

Investors could develop different techniques to divide active risk form systematic risk. This can be achieved by using derivatives, short positions and leverage, but the manager needs to be skilled in using such financial products in the right way if he is to achieve effective diversification.

It is important to note that the difference between leverage and risk is not clear. Most investors know perfectly well the meaning of risk, expected return, correlation coefficient and their roles in building an optimized portfolio. We can obtain such a portfolio by increasing the equity share, even if it is possible to increase (using leverage) the asset class characterized by low risk without increasing capital allocation in equity[15]. When we examine the relationship between risky assets and high return, we notice a linear path between risk and expected return. The difference between asset class returns decreases if we increase leverage in a selection of less risky asset classes, thus reducing the class of risky assets in the portfolio. For example, if we use leverage to increase government bonds, we build an asset class similar to equity as regards its risk/return relationship. This strategy may sometimes be analyzed incorrectly, because some investors believe that leverage increases risk. In actual fact, if leverage is used in portfolios with low risk assets, diversification is improved and risk reduced.

Jensen and Rotenberg[16] compose two different portfolios: the first with more equities than bonds and the second using leverage with less risky assets. The final results show that the leverage portfolio has less risk (5.3%) than the equity portfolio (10.3%) and equal return (10.2%) or higher return (13.4% against 10.2%) with equal risk (10.3%).

Markowitz's investor combines diversification with correlation, believing that diversification can be increased by using asset classes with low correlation. If an investor selected a correct mix of asset classes with low correlation and equal volatility, effective diversification could be achieved. The goal of equal volatility could also be reached if liquidity and risky assets were mixed; in this way the volatility changes without modifying the risk/return relationship or Sharpe ratio. The manager can select different assets with different risk profiles, while maintaining equal diversification. Risk scaling is used by hedge fund managers to improve the diversification level, in other words leverage is increased to improve the risk of strategies with low volatility and maintain liquidity to reduce the risk of strategies with high volatility.

Managers can obtain a mix of active risk and systematic risk if they select active positions on different assets and use derivatives of the same level of volatility. For example, Stock Index Futures can be bought to take a position on systematic risk and a long/short market neutral strategy can be

[15] Jensen G., Rotenberg J. (2004).

[16] Jensen G., Rotenberg J. (2004).

used to take a position on active risk. Some managers may concentrate portfolio cores on asset classes with active risk and keep the rest of the portfolio in traditional assets.

Grinold and Kahn[17] (2000) maintain that the information ratio depends on two factors: a) skill, in other words quality of information; b) breadth, that is the number of selections. Managers improve their performance by acting on the correct combination of the two factors. These conclusions have been integrated by other researchers[18], who maintain that expected returns may depend on managers' constraints. They consider that the equation for the expected return of a strategy is:

Information ratio = TC *IC*√N

TC = transfer coefficient is calculated as the correlation between the risk-adjusted expected returns and the risk weighted active exposure of securities in the portfolio. It measures the degree of information transfer from a security ranking signal into active portfolio weights for each security. A higher TC implies a more efficiently constructed portfolio, all else being equal. The transfer coefficient captures the effect of portfolio constraints on the expected information ratio (IR) of a portfolio strategy.

IC = the information coefficient is the expected correlation between predicted and actual return

N = number of independent securities to be chosen.

The transfer coefficient is lower than one and decreases to improve the constraints that are placed on the manager. Performance may depend on these constraints - in fact, a portfolio without constraints may have a value of close to one, while one with constraints may be equal to 0.3.

The effect of constraints is represented in the following table[19]

Table 1

Constraints	TC	TC percentage change
All constraints	0.332	
Industry	0.347	8.4%
Sectors	0.346	7.9%
Sectors and industry	0.298	-6.5%
M-cap	0.471	45.6%
Long only	0.678	108.1%

Source: Clarke-de Silva-Sapra

We notice that TC shows a lower value (0.33) when all constraints are imposed. This value in the table highlights that only 33% of information without constraints is transmitted into the portfolio.

4. Database and Methodology

Our dataset is composed of monthly data for the 5 asset classes included in table 2, for a total of 13 years and 3 months. In the following table we can see for each asset class the yearly return, yearly

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volatility and the Sharpe ratio according to the risk free rate. These data are illustrated on a return/risk Chart (figure 1) together with the efficient frontier of portfolios constructed with the same asset classes.

Table n.2

Realised Return and Risk in each asset classes (1/1994 - 4/2007)			
Asset class	**asset return**	**asset vol**	**sharpe ratio**
DJ EuroStoxx50	**10,2%**	**18,8%**	**0,35**
S&P500	**9,7%**	**14,1%**	**0,43**
MSCI US Govt Bond	**5,8%**	**4,5%**	**0,48**
MSCI Euro Govt Bond	**6,1%**	**3,8%**	**0,63**
MSCI US Govt Bond - 1y-3y	**4,8%**	**1,6%**	**0,69**
Risk Free (euro rate 3months)	**3,7%**	**0,3%**	**na**

The efficient frontier is built based on a traditional dataset for long only asset managers, that is maximizing the portfolio returns μ_ρ under the constraints of: a) volatility target σ_P; b) non-negative weight of each asset class ($w_i \geq 0$); c) a portfolio invested as follows: ($\sum_i w_i = 1$)

$$\sigma_P = (w^{|}\Omega\, w)^{\frac{1}{2}} = \sum_i w_i^2 \sigma_{ii} + \sum_i \sum_{j \neq i} w_i w_j \sigma_{ij}$$

$$\mu_P = w_i^{|} \mu_i$$

Max

s.t. target

$$w_i \geq 0$$

$$\sum_i w_i = 1$$

The efficient frontier is shown in Chart 1 below. The efficient frontier portfolio choices are shown in Chart 2 and in the following table (n. 3), according to their volatility.

Chart 1

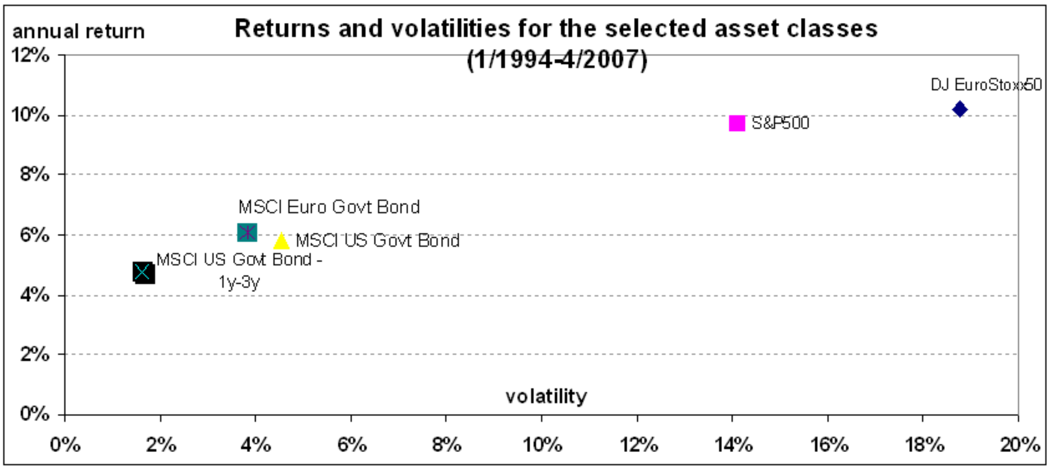
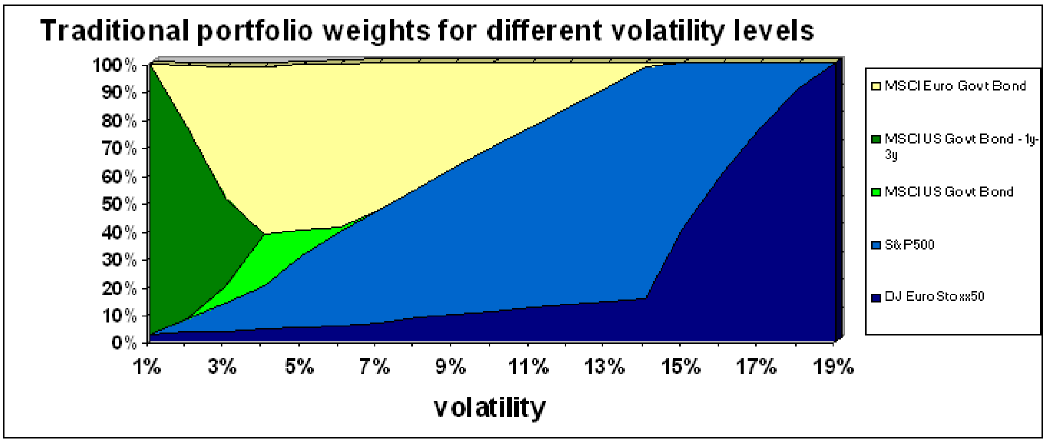


Chart 2



The aim of maximizing expected returns for each volatility level gives the following portfolios, characterized by the weight of equity, which varies between 3% and 100%.

The portfolios obtained are quite similar to the type proposed by financial managers: similar proposals to that shown in table 3 are frequently found in brochures for customers. These portfolios are characterized by a growing risk/return profile, which derives from increasing the equity share and decreasing the share of bonds. This aim is met when we move along the efficient frontier on the basis of risk, returns, and correlations.

A portfolio with an allocation of 40% equity and 60% government bonds is sold to the asset allocators as well-balanced and sold to clients as low-medium risk. However, as we can demonstrate, the risk is not diversified. Portfolio no. 6 shows a volatility level equal to 6%, 92% correlation with S&P500, and 37% correlation with bonds. R squared shows a more unstable situation from the point of view of volatility: in fact 84% of the portfolio's variance is explained by equities and only 14% by bonds.

Table 3

Portfolio n.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Volatilità	1,5%	2,0%	3,0%	4,0%	5,0%	6,0%	7,0%	8,0%	9,0%	10,0%	11,0%	12,0%	13,0%	14,0%	15,0%	16,0%	17,0%	18,0%	18,7%
CORRELAZIONI dei portafogli di frontiera con le singole asset classes																			
DJEurostoxx50	0,08	0,43	0,50	0,56	**0,72**	0,77	0,81	0,82	0,83	0,84	0,84	0,85	0,85	0,85	0,93	0,98	0,99	1,00	1,00
S&P500	0,17	0,52	0,61	0,68	**0,85**	**0,92**	**0,95**	0,97	0,98	**0,99**	0,99	0,99	0,99	0,99	0,95	0,89	0,84	0,80	0,77
MSCI US Govt Bond	0,81	0,66	0,55	0,50	0,28	0,15	0,07	0,02	-0,01	-0,04	-0,07	-0,09	-0,11	-0,12	-0,17	-0,20	-0,22	-0,23	-0,24
MSCI US Govt Bond - 1y-3	0,95	0,69	0,50	0,41	0,22	0,12	0,05	0,01	-0,02	-0,05	-0,07	-0,08	-0,10	-0,11	-0,16	-0,19	-0,21	-0,22	-0,23
MSCI Euro Govt Bond	0,57	0,72	0,75	0,71	**0,50**	**0,37**	**0,28**	0,20	0,15	**0,10**	0,07	0,04	0,01	-0,01	-0,02	-0,02	-0,02	-0,02	-0,02
R-QUADRO dei portafogli di frontiera con le singole asset classes																			
DJEurostoxx50	0,01	0,18	0,25	0,31	**0,51**	0,60	0,65	0,68	0,70	0,71	0,71	0,72	0,72	0,72	0,87	0,95	0,98	1,00	1,00
S&P500	0,03	0,27	0,38	0,46	**0,73**	**0,84**	**0,91**	0,94	0,96	**0,97**	0,98	0,98	0,98	0,98	0,90	0,80	0,71	0,64	0,59
MSCI US Govt Bond	0,66	0,43	0,30	0,25	0,08	0,02	0,01	0,00	0,00	0,00	0,00	0,01	0,01	0,01	0,03	0,04	0,05	0,05	0,06
MSCI US Govt Bond - 1y-3	0,90	0,48	0,25	0,17	0,05	0,01	0,00	0,00	0,00	0,00	0,00	0,01	0,01	0,01	0,02	0,04	0,04	0,05	0,05
MSCI Euro Govt Bond	0,32	0,52	0,57	0,50	**0,25**	**0,14**	**0,08**	0,04	0,02	**0,01**	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
ALLOCAZIONE dei portafogli di frontiera																			
DJEurostoxx50	3%	3%	3%	4%	5%	6%	7%	8%	10%	11%	12%	13%	14%	16%	40%	61%	77%	91%	100%
S&P500		5%	10%	15%	25%	33%	40%	47%	53%	59%	65%	71%	77%	83%	60%	39%	23%	9%	
MSCI US Govt Bond			6%	18%	9%	2%													
MSCI US Govt Bond - 1y-3	97%	69%	32%																
MSCI Euro Govt Bond		23%	48%	61%	60%	59%	53%	45%	37%	30%	23%	16%	9%	2%					
totale	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
% azioni	3%	8%	14%	20%	**31%**	**39%**	**47%**	55%	63%	**70%**	77%	84%	91%	98%	100%	100%	100%	100%	100%
% obbligazioni	97%	92%	86%	80%	**69%**	**61%**	**53%**	45%	37%	**30%**	23%	16%	9%	2%					

These data demonstrate how the risk diversification of traditional balanced portfolios is only apparent and a smokescreen policy on the part of the seller of the asset allocation. The data in table 3 shows the strong correlation with an equity portfolio, even with a medium-low level of volatility (from portfolio no. 5 upwards) . At a higher volatility level (for example portfolio no. 8, composed of 55% equities and 45% bonds) the dominance of risk from equities is overwhelming. The correlation with equities is equal to 97% and R squared of equities is 94%, compared to a portfolio variability from bonds of approximately 4% (even if bonds cover 45% of the whole portfolio).

We can also verify the drawdown dynamic of the portfolio analyzed compared to the SP&500 Index. We define drawdown as the "percentage loss of a portfolio's value from a past peak level"[20]. For example, we can suppose an initial portfolio value equal to 100. At the end of the first period (for example, one day or month) our portfolio value is 110. After n period its value is 90. After n+1 period the value is 105. Its maximum drawdown will be equal to (90-110)/110 = -18.2%.

From Chart 3 we can see that the drawdown of a balanced portfolio (no. 6 in table 3) is quite similar, regarding timing and maturity, to S&P500, albeit on a lower scale. The almost perfect temporal coincidence of the portfolio and SP&500 drawdown is sufficient to affirm that the risk of the balanced portfolio is not really diversified, but the investment seems similar to a deleveraged play on SP&500.

Chart 3

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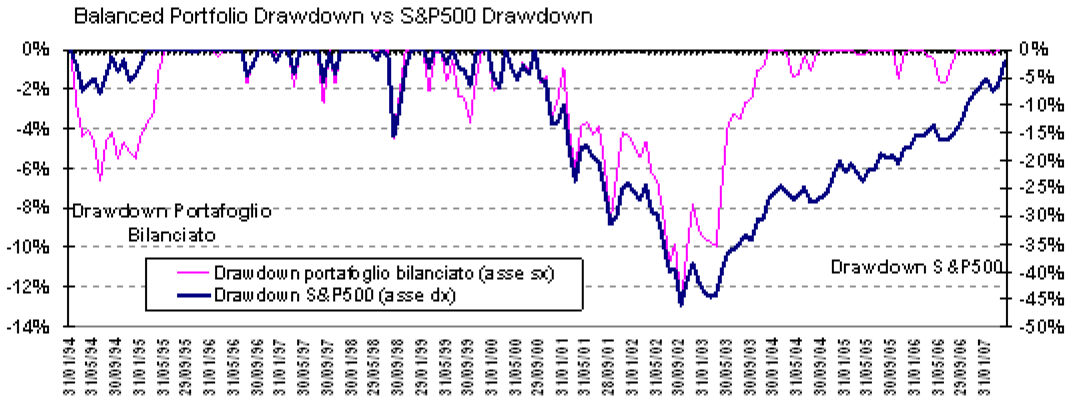


This evidence shows how the pie chart usually used to show portfolio capital allocation contains no information on the real degree of risk and gives no insight into risk contribution by the individual components of the portfolio. Efficient risk diversification in a portfolio should mean that, when portfolio drawdowns happen, the same drawdowns are not caused by the presence of a single asset that is drawdown-dominant compared to all other assets in the portfolio. The mistake of the financial industry lies in offering capital allocation diversification instead of risk diversification. If the aim in building a portfolio is to benefit from downside diversification, the traditional models of the asset management industry merely supply a pie chart of capital allocation in the various asset classes, without considering risk diversification.

This lack of consideration regarding risk allocation is clear from the fact that frontier portfolios usually show high concentrations of risk. Based on data from table 3, Chart 4 shows, for different levels of portfolios volatility on the efficient frontier, the percentage allocation of equities and bonds when the value of R squared is explained by the equity asset class. We can see that, even from medium-low volatility levels (5%-6%), portfolio variability described by equity components, is dominant and also high (R squared close to 80%).

If the decision variables in optimization processes are represented by portfolio weights, it is obvious that the final results will be linked to capital allocation. We will examine later how capital allocation should not be considered an important decision variable, but rather the result of an asset allocation process in which the decision variables are the individual risk allocation/contribution.

It is clear from the above that the risk allocation achieved is different from the perceived risk diversification obtained with capital allocation.

In this context the total risk of the portfolio is equal to the sum of the weighted marginal contributions to risk (MCTR) of the individual assets.

1) $$\sum_i w_i MCTR_i = \sum_i w_i \frac{d\sigma_p}{dw_i} = \sum_i w_i \frac{\sigma_{ip}}{\sigma_p} = \sigma_p$$

The total risk of the portfolio is not equal to the sum of the individual asset risks, but to the sum of the weighted marginal contribution to the portfolio risk. The marginal risk analysis provides the risk information related to the portfolio impact of a very small increases in each position in the portfolio (partial derivative of total risk in relation to individual asset weights), all other positions staying the same. We can obtain the correct breakdown of risk required to realize risk allocation with this marginal analysis.

Dividing Eq. n.1 by σ_p we have

$$2)\quad \sum_i \frac{w_i}{\sigma_p}\frac{d\sigma_p}{dw_i} = \sum_i w_i \frac{\sigma_{ip}}{\sigma_p^2} = \sum_i w_i\beta_i = 1$$

In this equation we show that the single percentages of contribution to risk (whose total sum is equal to 100%) is related to the betas β_i of single asset classes in relation to the portfolio. Each beta is weighted according to the weight of individual assets in portfolio w_i, in other words

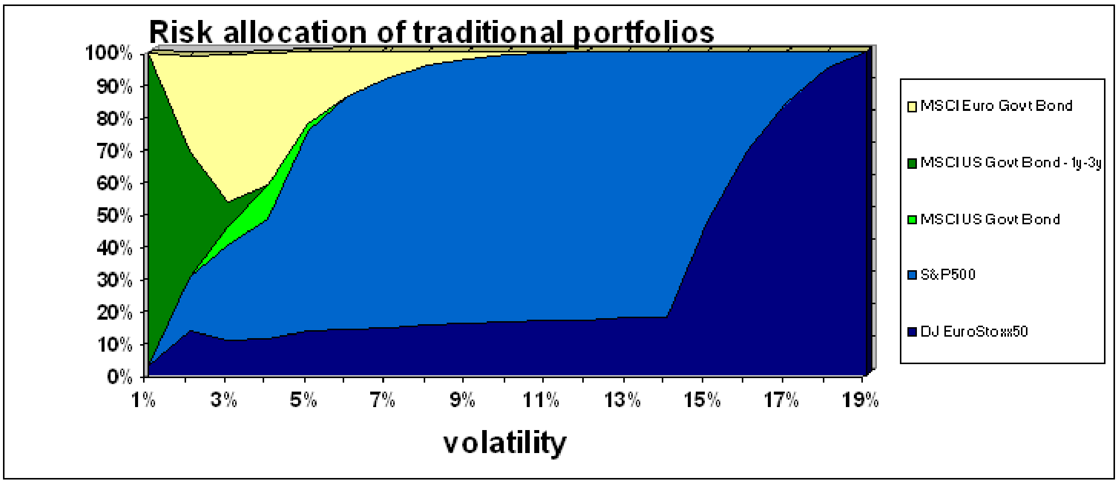
(3) $$PCTR_i = \omega_i\beta_i$$

Comparing the individual MCTR (i) to the total portfolio risk of equation (1) we can obtain the risk percentages of single asset classes in the portfolio:

(4) $$PCTR_i = \frac{w_i MCTR_i}{\sigma_p}$$

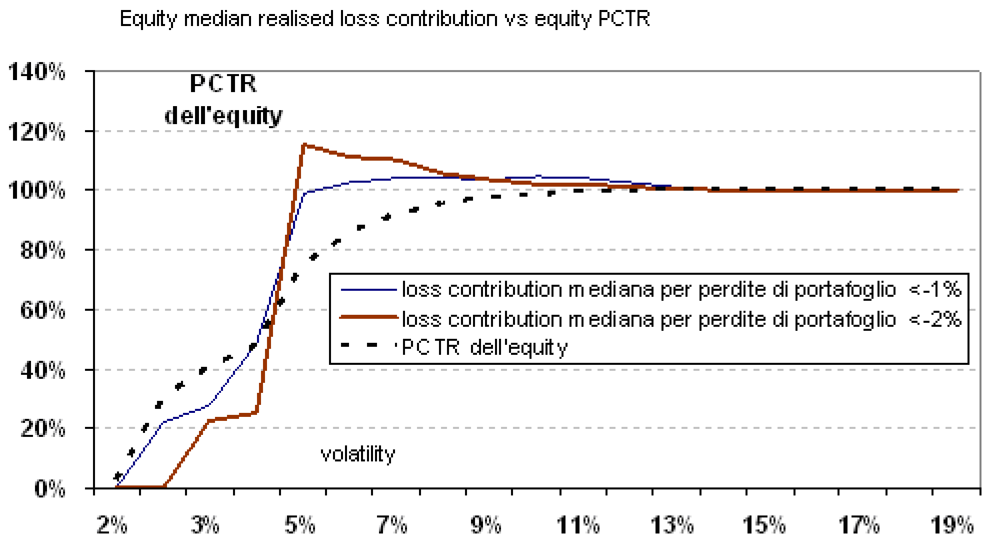
The percentage of total risk contribution shows that frontier portfolios concentrate on equity risk already at low volatility levels (Chart 5). The behaviour for different volatility level of this risk concentration can be observed the PCTRs shown in Chart 5.

Chart 4



Risk contribution (that is PCTR) gives a clear indication of the contribution to loss (Quian 2006) of each asset class to verify risk budgeting between the single asset classes of a portfolio. Using representative frontier portfolios, Chart 5 shows, according to the volatility of each portfolio, the median contribution of each equity asset class to this loss when there is a monthly portfolio loss greater than the registered threshold value (-1% and -2%). It is immediately clear that the PCTR of different portfolios provides an accurate indication of loss contribution. For example, suppose we invest in a balanced portfolio composed of 60% bonds and 40% equity: if there is a loss greater than 1% we observe that a percentage (100%) of equity contributes to this loss, against a forecast equity contribution of 86% (obtained by PCTR). This interpretation of PCTR helps explain the expected contribution to losses from the individual components of the portfolio and the importance of systematic diversification in the PCTR dimension in comparison to the weights of the portfolio dimension.

Chart 5



5. Asset allocation and marginal contribution to risk

Having examined the criticalities of the traditional approach to asset allocation, we analyze whether or not it is possible to build portfolios with more diversified risk allocation concerning available risk premiums (beta) and, if so, with what benefits and disadvantages compared to traditional methods. We hold that the aim of portfolio construction methods should be to obtain diversified portfolios from the downside point of view.The decision to support each systematic risk in a portfolio derives from the manager's skill in extracting risk premium (extra returns in relation to the risk free rate). We will also verify how systematic risk diversification contributes to improving the diversification of the source of portfolio returns, so that each risk premium contributes to the total portfolio return *independently of its risk level.* Such a contribution of all risk premiums to portfolio returns makes this type of portfolio more stable and less volatile than traditional portfolios. Downside diversification, obtained in a constant manner and independently of the portfolio risk level, contributes to reducing the intensity and length ("time under water") of drawdowns. Using PCTR and a given portfolio risk level, we can determine the percentage weights so the PCTR of individual assets in the portfolio is similar, that is

(5) $$PCTR_i = \frac{1}{N}$$

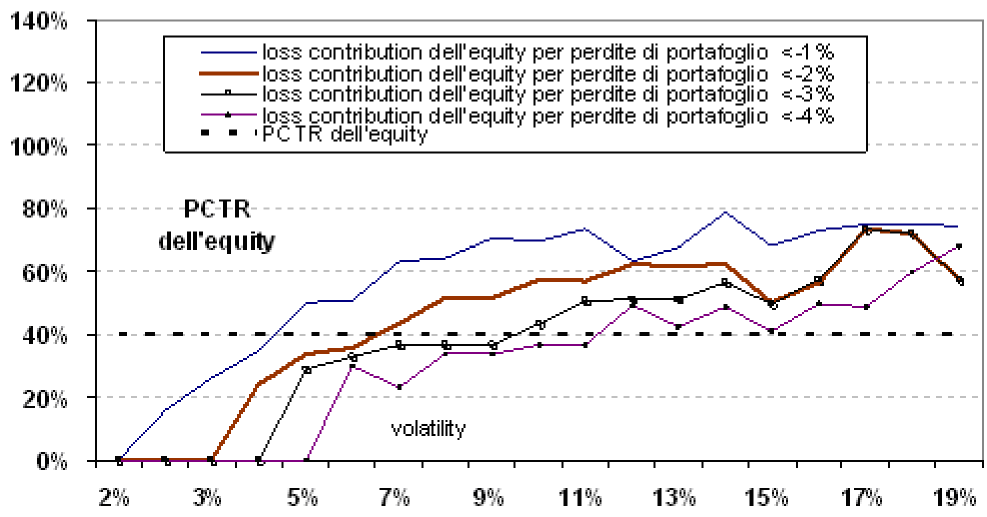
where N indicates the number of asset classes.

We can define this portfolio construction as Equal PCTR.

We constructed 19 different Equal PCTR, with similar volatility to 19 traditional efficient portfolios (table 3). An analysis of Chart 7 shows the economic significance of PCTR and the importance of risk diversification obtained by this method. In fact, increasing portfolio risk (volatility rises by 1.5% to 19%) and the effective loss threshold (which measures the risk contribution of equity) by -1% to -4%, equal PCTR portfolios show a decreasing contribution to these losses. This situation converges towards equity PCTR, which is obtained from the equal PCTR approach (equity PCTR is equal to 40% of the total risk and is represented by a horizontal dashed line in the Chart). This approach is able to ensure risk diversification when we expect great losses in a portfolio.

Chart 6

Equal PCTR portfolios: equity realised loss contribution vs equity PCTR



Risk diversification is obtained independently of aggressive choices, in other words we /benefit from diversification at any volatility level selected in the portfolio. We can compare equal PCTR risk allocation (Chart 7) and traditional frontier portfolios (Chart 4). It is clear in the traditional approach that frontier portfolios show high risk concentration along the efficient frontier. This concentration is absent in the equal PCTR approach. Using five asset classes, risk allocation (PCTR) is equal to 20% for each class, independently of the volatility level of the portfolio.

Chart 7

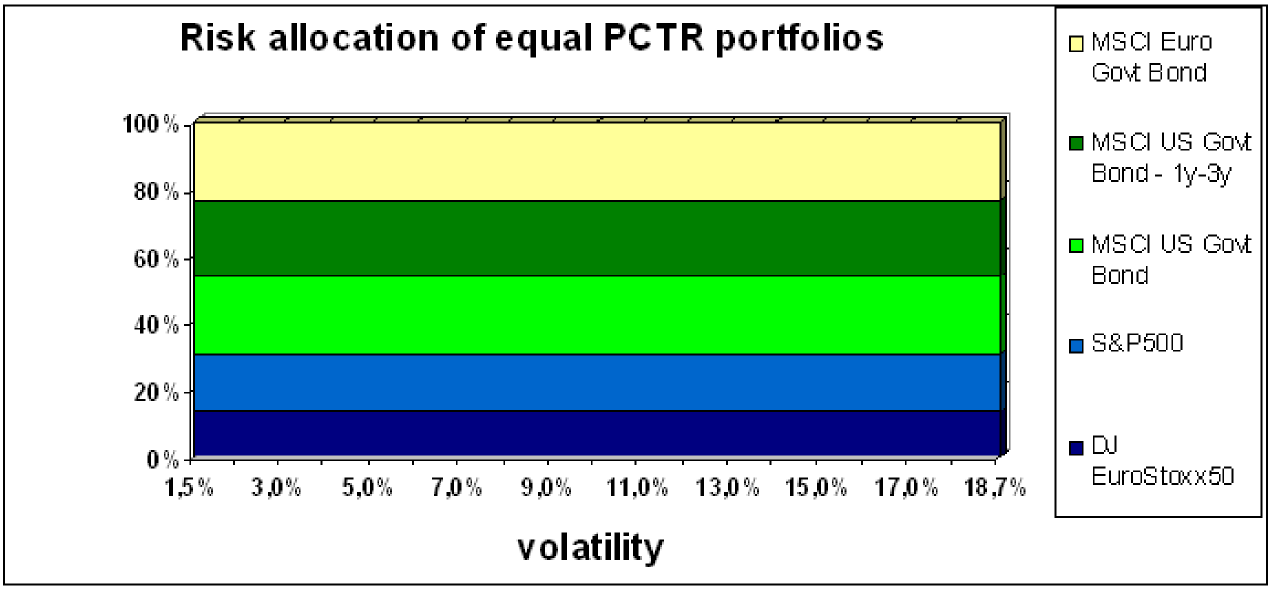
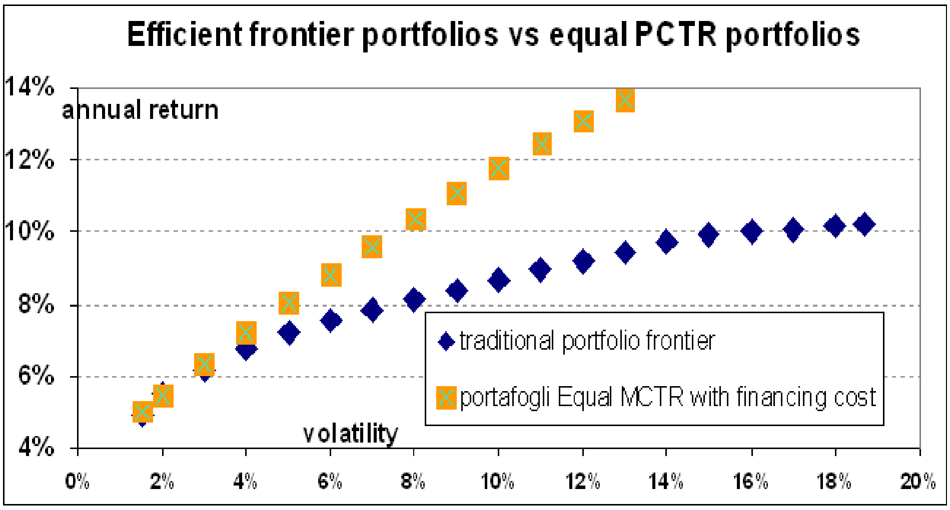


Chart 8 shows a comparison of risk/return between the two construction methods. We notice a reduction of volatility with equal PCTR portfolios, with the same return. In the absence of leverage constraints, diversification doesn't decrease (that is it doesn't concentrate on a single beta such as the equity beta) with increased risk, but the portfolio is diversified between all possible beta exposures. With the data described, leverage is used for each portfolio characterized by a volatility level greater than 3%.

Chart 8



An important result of diversification obtained by using all available systematic risk premiums (derived from an equal PCTR approach) is the effective mitigation of drawdowns in relation to traditional portfolios characterized by equal volatility. Equal PCTR portfolios show max drawdowns that stabilize at levels equal to 2 times the volatility (see table 4), while traditional portfolios show max drawdown values 3 times higher than volatility. We notice that the max drawdown-volatility ratio seems more stable in the equal PCTR approach than in the traditional approach.

Table 4

Volatility	Drawdowns		Ratio Max Abs Drawdown \ Volatility	
	Traditional portfolios	Equal PCTR portfolios	Traditional portfolios	Equal PCTR portfolios
1.5%	-1.4%	-1.6%	0.93	1.05
2%	-2.7%	-2.5%	1.33	1.27
3%	-4.5%	-4.5%	1.48	1.51
4%	-6.0%	-6.5%	1.50	1.62
5%	-7.0%	-8.4%	1.41	1.68
6%	-13.0%	-10.6%	2.17	1.76
7%	-18.6%	-12.8%	2.65	1.82
8%	-23.6%	-15.0%	2.95	1.87
9%	-28.3%	-17.2%	3.14	1.90
10%	-32.6%	-19.3%	3.26	1.93
11%	-36.6%	-21.4%	3.33	1.95
12%	-40.4%	-23.5%	3.37	1.96
13%	-44.0%	-25.6%	3.38	1.97
14%	-47.3%	-27.6%	3.38	1.97
15%	-51.3%	-29.7%	3.42	1.97
16%	-54.7%	-31.6%	3.42	1.98
17%	-57.5%	-33.6%	3.38	1.97
18%	-60.0%	-35.5%	3.34	1.97
19%	-61.6%	-36.9%	3.24	1.97

The greatest diversification between risk premiums gives significant improvements in the downside, and smaller drawdowns, with equal volatility compared to a traditional portfolio. This empirical observation is different from the parametric relationship, utilizable only in normal distribution. In the latter case one-to-one mapping between volatility and drawdown can be performed according to the threshold of expected return (Belentepe, 2003; Atiya et al., 2003e). This parametric approach has been applied to the analysis of hedge fund strategies (De Prado & Peijan, 2004), where drawdown distribution analysis is much more used than in traditional asset management.

6. Conclusion

In this article we have examined a weakness of traditional methods of optimum portfolio construction: the risk concentration of asset allocation. The concentration of risk is clear for large parts of the efficient frontier. If the aim of portfolio construction is to obtain real diversification (i.e., downside risk diversification), the most direct approach is based on the estimation of the contribution to total portfolio risk of each class of assets. This result is important because it gives an insight into the contributions to loss of individual asset classes when there are relevant losses in the portfolio. Equal PCTR portfolios offer high diversification of risk contributions and have lower levels of parameter estimation errors, because they are “only” exposed to estimation errors of the variance-covariance matrix and so it is not necessary to estimate expected returns. These portfolios reduce overexposure to a specific asset class and at the same time offer good exposure to all individual asset classes. In this way they provide diversified allocation and return/risk contribution with the same expected return or risk as traditional portfolios. This result is supplied by each risk premium on the market and/or used independently by the volatility level of the portfolio. Risk diversification is constant for the selected risk level. We have verified how greater diversification among risk premiums provides consistent downside improvement with smaller drawdowns compared to portfolios built according to the traditional mean-variance approach.

Diversification in the PCTR dimension provides a new asset allocation approach whereby, given a target portfolio’s risk level, “no skill weights” or “neutral weights” can be represented by the set of weights that delivers a risk budgeting characterized by maximum diversification of PCTR, and consequently of the portfolio downside risk. In this framework the percentage weights of each assets in the portfolio are a consequence of risk allocation rather than the result of portfolio optimization, which only works on the total risk of the portfolio.

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