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Risk Perception and Bootstrap CDAR

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1.1 The risk management framework

- **Risk Analisys ecompasses 3 interrelated elements (Kunreuther, “Risk Analysis2”):**
  - **Risk assessment**
    - RA studies estimate the chance of a specific set of events occurring and/or their potential consequences
  - **Risk perception**
    - RP studies concerned with the phychological and emotional factors that have been shown to have an enormous impact on behaviour
  - **Risk management**
    - RM studies concerned the strategies for reducing future losses keeping in account the two abovementioned elements
      - **Need for developing strategies that involve risk communication, economics incentives, standards and regulations for managing these risk**

→ Need for risk communication? Is there a communication risk? How’s the role of the risk perception? Is risk perception really important for the positioning of our industry?

1.2 A step back

- **Paul Slovic on human intellectual capabilities:**
  - **William Shakespear vs Herbert Simon**

- **Glyn Holton on "defining risk"**
  - **subjective vs objective interpretation**
    - Finetti vs Frank Knigth vs Markowitz
      - **Finetti said "probability doesn't exist**
      - **Knight defined risk as the measurable uncertainty**
      - **Markowitz offered no definition of risk; …"the investor does consider expected return a desiderable thing and variance of return an undesiderable thing…"**

      → **Since risk is considered as exposure and uncertainty, it is a condition of individuals that are self-aware**

- **What Operationalism said on risk?**

1.2 A step back

- **Operational Definition:**
  - **Percy Bridgman in 1927 said in "The logic of Modern Physics" as reported in Holton:**
    - "…if all knowledge of the world stems from our experiences, then definitions can be meaningful only if they refer to experience…we formally define a concept by specifying a set of operations through which that concept is experienced…in general, we mean by ay concept nothing more than a set of operation: the concept is synounymous with the corrisponding set of operations return a desiderable thing and variance of return an undesiderable thing…"

→ **Since is impossible to define exposure and uncertainty in term of "that which can be perceived", it is impossible to operationally define risk.**

→ **At best, we can operationally define our perception of risk! There is no true risk.**

→ Holton conclusion: "…It is meaningless to ask if a risk metric capture risk. Instead, ask if it is useful…"

1.3 A step forward

- **In order to answer to Holton conclusion we need to know:**
  - **What clients/stakeholders want?**
  - **Are your clients rational actors or rational fools?**
  - **And…..**
  - **We need to understand if what we usually use is coherent with all abovementioned:**
    - if yes, good job mate!!
    - If not, how we can face this challenge?

1.3 A step forward

Since risk perception is “on the air” what we can say about a self-aware individual behaviour?

Let’s looking at the “use of risk perception” keeping in mind the definition:

- **The most important theories are:**
  - SP/A Theory (Lopes 1987)
  - Prospect Theory (Kahnemam and Tversky, 1979)
  - FS Puzzle (Friedman and Savage, 1948)
  - Behavioural Portfolio Theory (Statman, 2002)

- **which is the LCD of the firt three theories?**

→ Safety-First Portfolio (from Roy to Kataoka,Telser, Arzac and Bawa)

1.3 A step forward

- **Roy's SFPT, let:**
  - **W: terminal wealth**
  - **s: subsistence level** $\text{Probability of ruin} = \min \Pr(W < s)$

    → an investor is ruined when his terminal wealth falls short of a subsistence level s

- **According to Kataoka SF investor aim to maximize the subsistence level subject to the constraint that the probability that wealth (W) falls below the subsistence level (s) does not exceed a predetermined** α

- **According to Telser a portfolio is considered safe if the probability of ruin doesn't exceed** α **An investor choose a portfolio to maximize expected wealth E(W) subject to**

$$\Pr = \{W \leq s\} \leq \alpha$$

- **According to Arzac and Bawa one can extend Telser's model by allowing** α **to vary. In particular in a choice over** $[E(W), \alpha]$ **pairs, the utility function *u* is defined by:**

$$u(W) = W \quad \text{if } \Pr\{W < s\} \leq \alpha$$

$$u(W) = W - c \quad \text{if } \Pr\{W < s\} \geq \alpha \qquad \text{with } c > 0$$

1.3 A step forward

- **Have a look now on Lopes SP/A Theory. Let:**
  - **S: security**
  - **P: potential**
  - **A: aspiration**

In the Lopes' framework fear and hope function altering the relative weights attached to decumulative probabilities. For example, in a two-date framework,Let's there be n states associated with date one where $p_i = \Pr[W_i], i = 1,2,\ldots n$ **. The expected wealth** $E(W) = \sum p_i W_i$ **can be expressed as** $\sum D_i (W_i - W_{i-1})$

→ **Fear operates through an overweighting of the probabilities attached to the worst outcome relative to the best outcomes.**

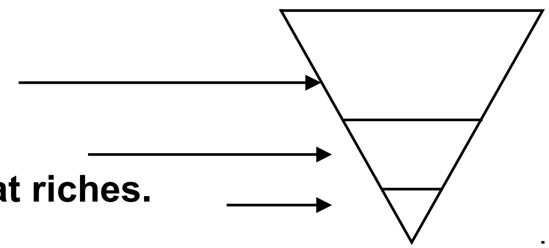
→ **On the contrary hope leads individuals to act as if they were unduly optimistic when computing E(W).**

1.3 A step forward

- **Meier Statman**
  - **Investors want more than protection from poverty; they want to be riches as well.**

- **According to Statman, how investors should build their portfolios according to the previous statement?**

  → **LAYERED PYRAMIDS**
  - **bonds in the bottom layer for protection from poverty**
  - **stock mutual funds in the middle layer for moderate riches**
  - **individual stocks and lottery tickets in the top layer for great riches.**



- Cohen state that one's position in the hierarchy very much relates to how much control you have over your life and your opportunity for full social engagement.

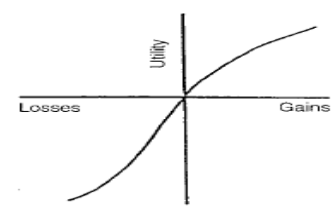
- The most important risks (to be) faced by investor are both loss of wealth and decline in status, not variance in return!!

1.4 And now?

- **Tversky and Wakker on the composition of risk preference and belief**
  - **Under subjective expected utility, we have a clear separation of value and belief. The utility function is taken as measure of value and the probability measure is naturally interpreted as a measure of belief. Belief is indipendent of decisions since the probability of an event does not depend on the consequences attached to it which may be taken as a desideratum for belief.**

- Are expected utilities stable over time?
  - Not at all…and....
    - Expected utilities are path dependent!! So….

1D. Kahneman and Tversky's Prospect Theory Utility Function



- Risk management "works" against a "mutant"

  → The challenge is to use something that can minimize the probability of possible future changes in the initial conditions of risk perception

1.4 … we do it our way.

- Let’s see our “weapons”:

$$VaR_{\alpha,\Delta T} := \left\{ \varphi_{dt} \,\middle|\, \int_{-\infty}^{\varphi_{dt}} \Psi(x) \cdot dx = \alpha \right\}, dt = \Delta T > 0$$

- According to aformentioned, whats’s “wrong” with that?
- What do we need to change?

$$DD_{\alpha,\Delta T} := \underset{dt}{Max} \left\{ \varphi_{dt} \,\middle|\, \int_{-\infty}^{\varphi_{dt}} \Psi(x) \cdot dx = \alpha \right\}, \forall\, dt > 0$$

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2.1.1 Definitions

- **Some rules for Coherence**

- **Assume a measure ρ, uncertain value X,Y, certain values $\lambda, \alpha > 0$**
  - **Positive Homogeneity** $\rho(\lambda X) = \lambda \rho(X)$
  - **Translational Invariance** $\rho(X + \alpha) = \rho(X) - \alpha$
  - **Sub-additivity** $\rho(X + Y) \leq \rho(X) + \rho(Y)$

→ **If the premium doubles, than the risk measure doubles too.**

→ **If we add (subtract) to initial risk position a certain value (α), risk measure increase (decrease) accordingly.**

→ **The last is the diversification principle. “A merger does not create extra risk”**

2.1.1 Definitions

- **Given the R.V. *X*, which represent on a given holding period the economic consequence oa a decision, - e.g. porfolio return - ,**

  → **VaR (at level)** α **is defined as:**

$$VaR_{\alpha}(X) := \sup\{x \mid \mathbb{P}[X \leq x] \leq \alpha\}$$

where $\alpha \in (0,1)$

- **If we have the inverse cdf for *X* it is possible to give a simple VAR expression. In this case, being**

$$F_X(x) := \mathbb{P}[X \leq x]$$

$$VaR_{\alpha}(X) := F_X^{-1}(\alpha)$$

2.1.2 Economics meanings

- **In a portfolio environment Var could be intended as:**

  - **Minimum loss the portfolio can suffer on the given time horizon in the A% (e.g. 5%) worst cases**

  OR

  - **Maximum loss the portfolio can suffer on the given time horizon in the 1-A% (e.g. 95%) best cases.**

- **Previous definitions is a proper one when *X* express a revenue; when, on the contrary the growing of *X* means a loss, we have:**

$$VaR_{\alpha}(L) = \inf\{l \mid P[L > l] \leq \alpha$$

2.1.3 Limits

- **VaR is relatively simple to implement and can be applied to several kind of risk, but it shows some strong limitations on both the conceptual and methodological sides.**

- **VaR is not sub-additive: in fact, it might happen that VaR calculated on an aggregated position is greater than position 's components VaR. Therefore it is not suitable for portfolio selection, since it is inconsistent with diversication tenet and can be dangerous in the contest of capital allocations as well (with disjoint risk units).**
  - → **These limitations are so strong that many do not consider it a risk measure at all!**

- **Besides the VaR measure does not explain to us what happen in those case, despite being infrequent, when "worst cases" happen. Think of default event: our model can assess the PD as being very rare, but we still need to know the Loss Given Default.**

2.1.4 Adressing the shortfall area

- **There are several measures that try to address the lack of knowledge of the shortfall area beyond VaR, that is worst cases scenarios.**
  - **Among these we have the Expected Shortfall and Conditional Value At Risk.**

- **The Conditional Value At Risk is a conditional expectation of losses, in which the condition is that the loss is included in the shortfall area.**
  - **There are at least two versions of CVaR:**
    - Mean Excess Loss or Mean Shortfall:

$$CVaR^{+}(X) = E\left[X \middle| X < Var(X)\right]$$

    - Tail Value at Risk

$$CVaR^{-}(X) = E\left[X \middle| X \leq Var(X)\right]$$

2.1.4 Expected shortfall et al.

- **Expected shortfall is substantially similar to CVAR:**

$$ES_\alpha(X) = -\frac{1}{a}\left(E\left[X\mathbf{1}_{\{X \le Var_\alpha(X)\}}\right] - Var_\alpha(X)\left(P\left[X \le Var_\alpha(X)\right] - \alpha\right)\right)$$

→ **In this case first term is still the expected value losses over the shortfall area.**

→ **The second term adjust this value with difference between worst cases probability and confidence level.**

- **Conditional VaR and VaR**
  - **For CVar own nature of loss level beyond VaR, it must necessarily be:**

$$CVaR_a \le Var_a$$

- **Expected Shortfall and Conditional VaR**
  - **In standard conditions, that is for continuous functions, the confidence level is the very shortfall probability, i.e** $P\left[X \le Var_\alpha\right] = \alpha$
    - It can be shown that Expected Shortfall is given by Mean Excess Loss ($CVaR^+$)

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2.2.1 Defining DD Measures

- **Drawdown measures are extremely intuitive and can account for portfolio loss over a defined time horizon. Contrary to other measures, for Drawdown loss event is not unique, but it is the sequence of losses over the given time horizon that concur to build the drawdown.**
- **Assume a portfolio with n instruments and take:**

$$r(w,t) \quad with \quad w \in R^n, \sum w_i = 1$$

as the logarithmic return obtained investing, on *[0; t]*, in each single *n* instrument respectively the w_i share of W_0 wealth.

→ **Note that, given *r(w; t)* return with reference to time horizon *[0; t]*, we can think of innite intermediate values r(w;) varying on in [0; *t*]. By comparing those values, in the following, we get to denition of drawdown.**

2.2.1 Defining DD Measures

- **Drawdown is given by difference between the maximum *r(w; τ)* intermediate returns one can obtain in *[0; t]* and the final return *r(w; t).***

$$DD_{wt} = \max_{\tau \in [0,t]} r(w,\tau) - r(w,t)$$

- **Two related variables are the Maximum Drawdown:**

$$MD_{wt} = \max_{\tau \in [0,t]} DD_{w,t}$$

- **And the Average Drawdown**

$$AD_{wt} = \frac{1}{t}\int_0^t DD_{w,t}\,d\tau$$

2.2.2 DD and the Conditional DAR

- **DaR definition is very similar to the VaR one, nevertheless in this case target variable is not gain/loss, but the maximum drawdown in the holding period.**

$$DaR_{\alpha}(MD) = \inf\{d | P[MD > d] \leq \alpha\}$$

- **The Conditional Drawdown at Risk can now be defined as:**

$$CDaR_{\alpha}(MD) = E[MD | MD > Dar(MD)]$$

→As for CVaR case, we have a conditional expectation, but now condition is expressed by the worst cases event, that is by the the event that maximum drawdown is located over DaR.

2.2.3 Time Under Water

- **Time Under the Water expresses the period of time our risky investment may have returns under its historic record mark. The Time Under the Water is computed as:**

$$TUW_t = t - \underset{\tau \in [0,t]}{\arg\max}\, r(w,\tau)$$

- **We can compute a maximum TUW over give period**

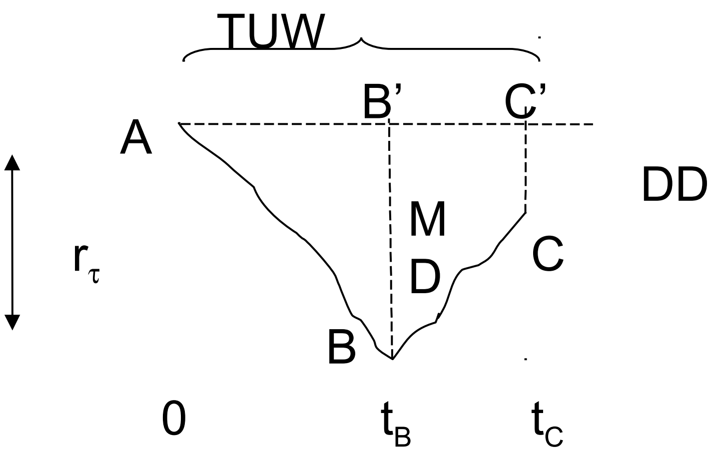
$$MT = \max_{\tau \in [0,t]} TUW_\tau$$

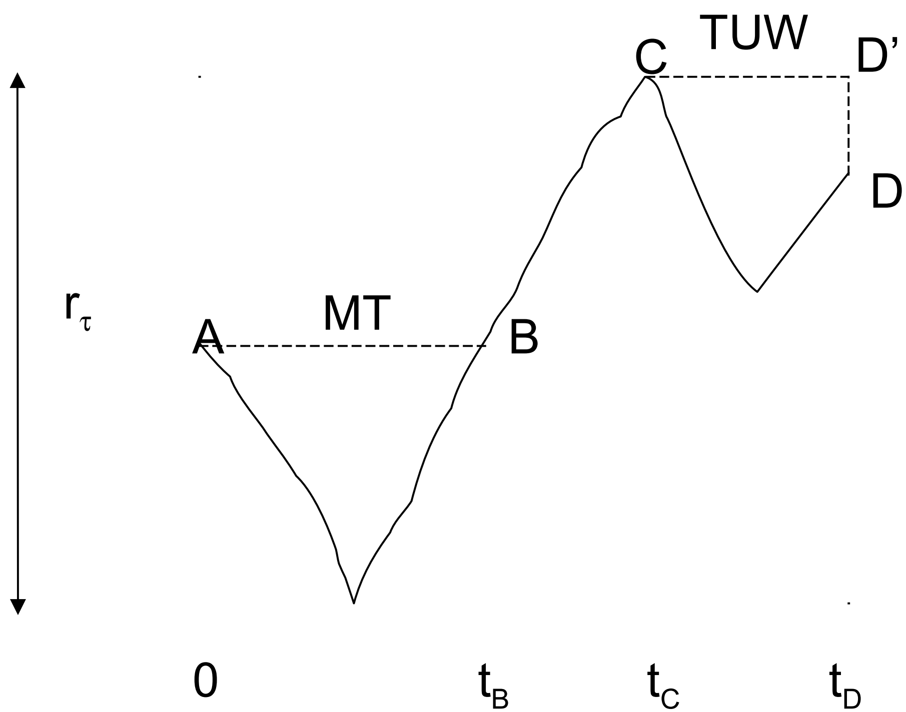
- **And assess a confidence level of being the time over the maximum threshold:**

$$TaR_\alpha(MT) = \inf\{t \mid P[MT > T] \leq \alpha$$

2.2.4 Watching CDAR and TUW

- **Let’s see graphically the CDAR and TUW**





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3.1 Intro

- **Let's see a typical Investment Management Agreement:**

  - "...The objective is to produce a return from inception of 1% per annum above the benchmark, subject to a minimum time period of three years. The return will not fall more than 3% below the benchmark in any (time during a) 12-months interval".

- **Is it possible to find a risk measure directly written on the above specified risk perception?**

- **Keep in mind that:**
  - **In calculating (value-at) risk an instantaneous price stock equivalent to a 10 days movement in prices is to be used (Basle Committee,1996)**

→ What we can do to be compliant with all abovementioned?

3.2 The Relative CDAR distribution

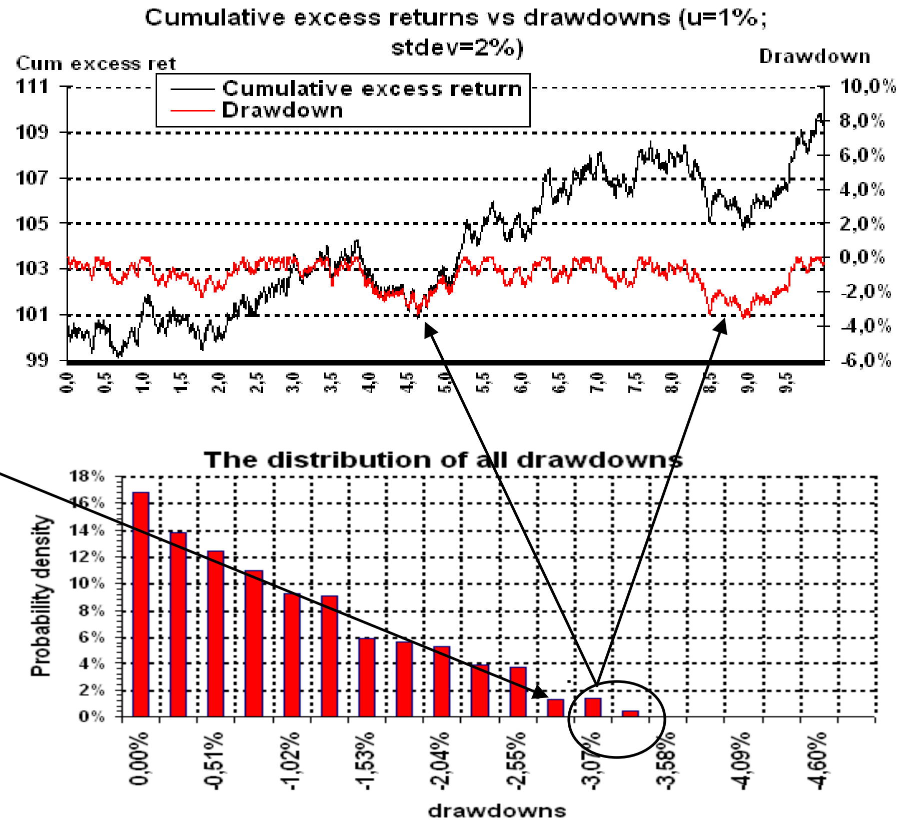
- **Focus of our analysis is the Cumulative Excess Return vs Benchmark**
  - If the measure the CDAR on the cumulative excess return we deal with a different measure called Relative CDAR.

- **This measure allow:**
  - A straightforward way of managing the client's (in-)tolerance to the potential of lower-than-the-benchmark performances (reputational risk)
  - ...by construction, the calculation of drawdowns is made from an "high water-mark" (relative drawdown= distance from the previous high water-mark) (performance fee;economic risk)
  - Drawdown is the only risk measure directly built on long term horizons without the needs of both scaling factor &/or overlapping returns

3.2 The Relative CDAR distribution

- **Let's see an example of Cumulative Excess Return vs Benchmark (left axis) and the corresponding drawdown behaviour (right axis) and the related drawdown density function**

→ Focus on R-CDAR (99%)

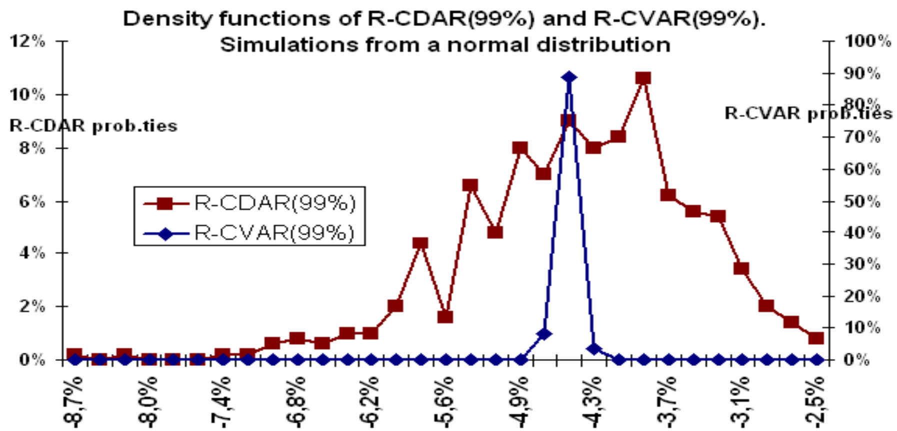


3.2 The Relative CDAR distribution

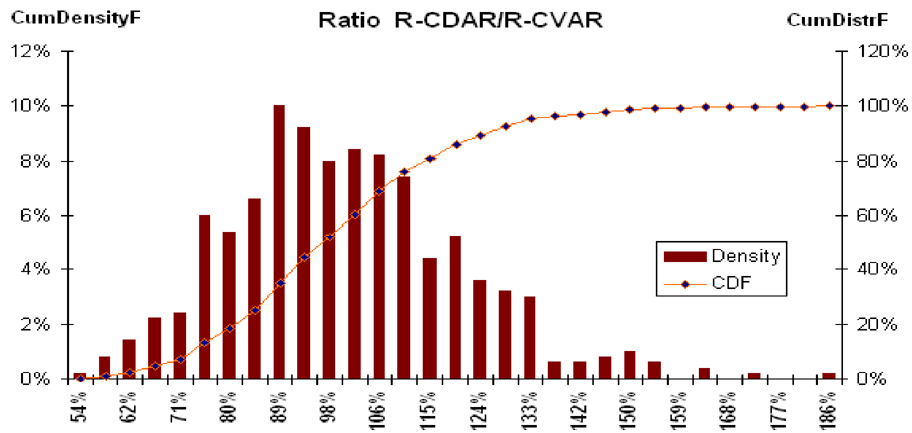
- **Have a look at the Relative–CDAR (99%) distribution in case of normal distribution**

→ Wide dispersion of R-CDAR values make estimation in the drawdown world a much more challenging task

→ 500 sim, N(0;2%)

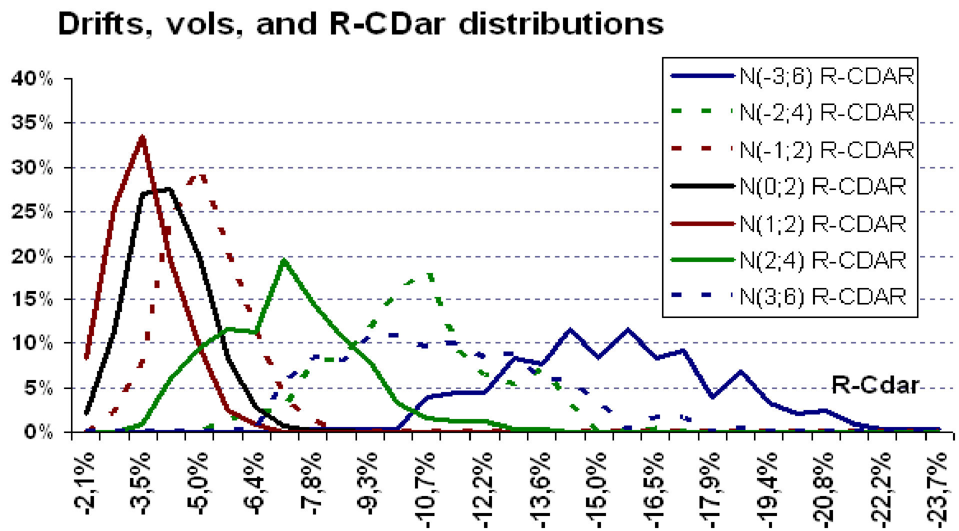


→ The ratio R-CDAR / R-CVAR is between 54% and 180% in this simulation



3.2 The Relative CDAR distribution

- **What happens with different drifts and volatilities?**

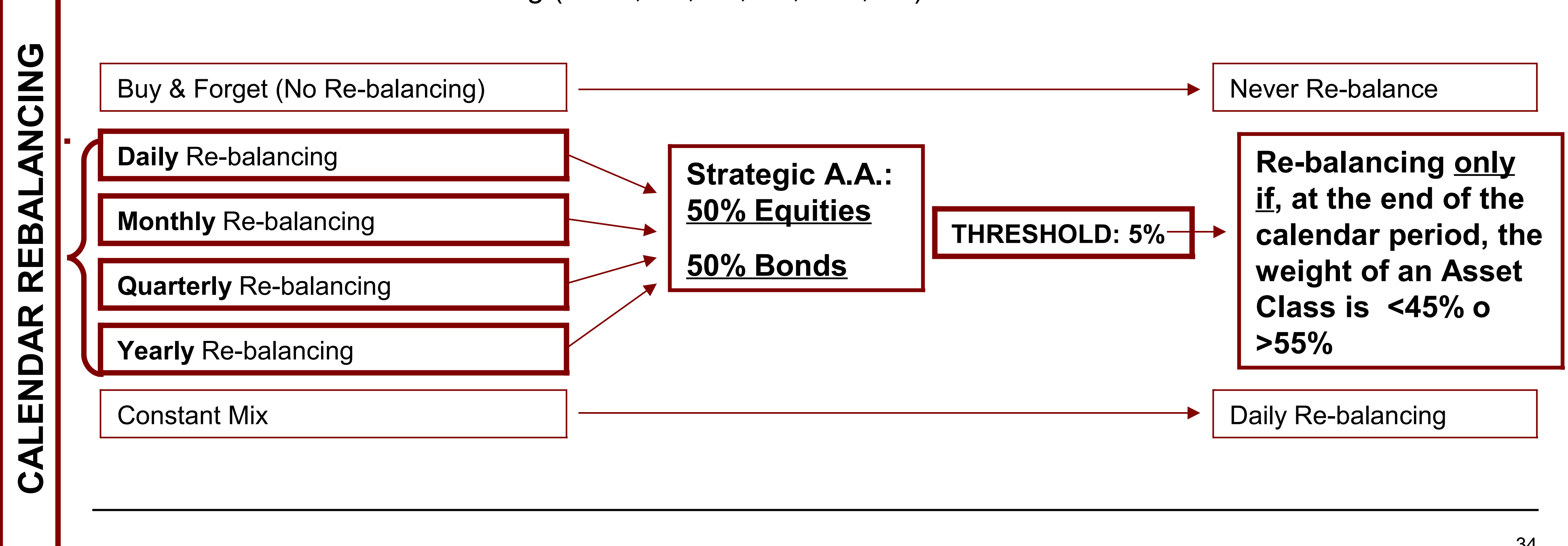


→ Higher volatilities = bigger uncertainty in R-CDAR (99%)

→ Drift effect: a (soft) location parameter (no overlapping in the R-CVAR case)

3.3 R-CDAR vs R-CVAR

- **Now we will analyse a real-life portfolio problem. In this example we let the weights drifts as in real life portfolios, subject to a prespecified portfolio re-balancing disclipline.**
  - **Benchmark 50% equities, 50% bonds**
  - **16 simulated active porfolios: Bond active weights from –30% to +30%**
  - **Re-balancing discipline: semi-constant active mix obtained via:**
    - **Calendar** re-balancing (daily, monthly, quarterly, yearly)
    - **Threshold** re-balancing (+-5%;+-2,5%;+-1,25%;0%)



(*) 6yrs of historical daily data: 1999-2004

3.3 R-CDAR vs R-CVAR

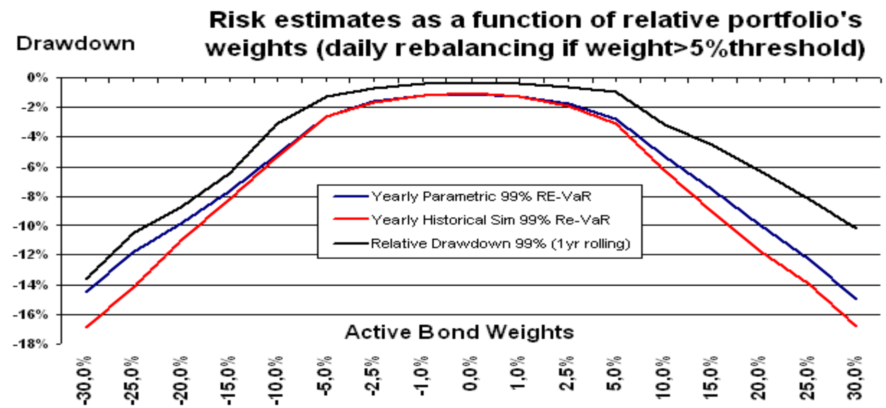
- **Hyp: the drift term is unknown**

→ **Both R-CDAR and R-CVAR get bigger with higher level of active risk**

→ **R-CVAR is always more conservative than R-CDAR**

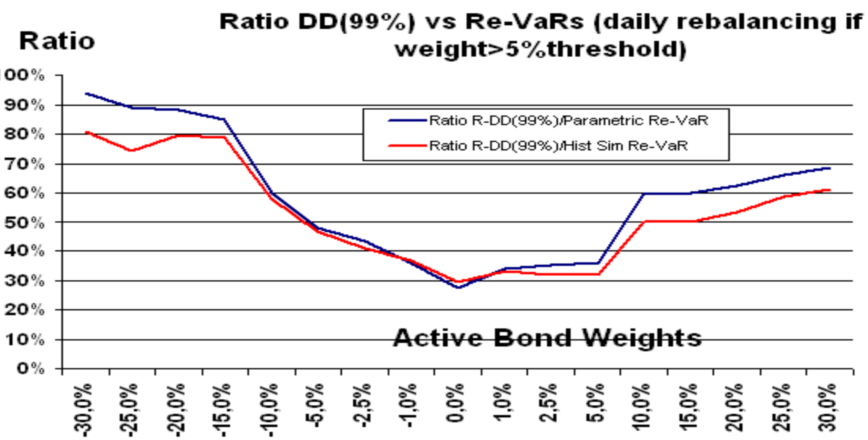
→ **Conservativeness of R-CVAR is exacerbated in low active risk level environments**

→ **No stable relationship between the two**



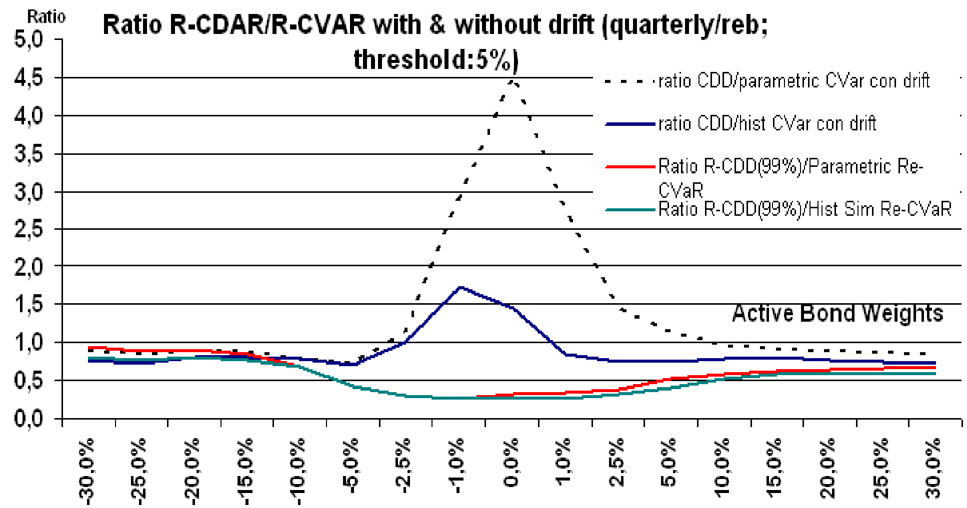
Quarterly Rebalancing (only if A.A. weights are over the specified threshold: +-5%)

Active Bond Weights	Whole Sample Over/Under performance	Yearly Parametric 99% RE-CVaR	Yearly Historical Sim 99% Re-CVaR	Yearly Relative CDrawdown 99%	Ratio R-CDD(99%)/ Parametric Re-CVaR	Ratio R-CDD(99%)/ Hist Sim Re-CVaR	yearly excess performance
-30,0%	-5,3%	-14,2%	-16,7%	-13,5%	95,3%	81,2%	-1,1%
-25,0%	-3,2%	-11,5%	-13,6%	-10,3%	89,3%	76,0%	-0,6%
-20,0%	-0,6%	-9,3%	-10,3%	-8,4%	90,0%	81,0%	-0,1%
-15,0%	2,0%	-6,9%	-7,5%	-5,8%	83,4%	77,0%	0,4%
-10,0%	3,1%	-4,7%	-4,7%	-3,2%	67,7%	67,7%	0,6%
-5,0%	5,2%	-2,5%	-2,5%	-1,1%	42,6%	42,3%	1,0%
-2,5%	6,2%	-1,7%	-1,7%	-0,5%	31,0%	30,1%	1,2%
-1,0%	6,8%	-1,5%	-1,6%	-0,4%	26,9%	25,3%	1,3%
0,0%	7,1%	-1,5%	-1,7%	-0,5%	32,3%	28,1%	1,4%
1,0%	7,5%	-1,7%	-2,2%	-0,6%	34,9%	27,1%	1,5%
2,5%	8,0%	-2,1%	-2,7%	-0,8%	39,1%	31,2%	1,6%
5,0%	8,8%	-3,1%	-3,9%	-1,6%	51,4%	41,1%	1,7%
10,0%	11,5%	-5,6%	-6,4%	-3,3%	59,0%	51,7%	2,2%
15,0%	12,6%	-7,9%	-8,6%	-5,0%	63,0%	57,5%	2,4%
20,0%	13,7%	-10,2%	-11,4%	-6,7%	65,2%	58,6%	2,6%
25,0%	14,4%	-12,5%	-14,1%	-8,4%	67,1%	59,8%	2,7%
30,0%	17,2%	-15,2%	-17,2%	-10,1%	66,4%	58,7%	3,2%



3.3 R-CDAR vs R-CVAR

- **Hyp: the drift term is known**
- **Adding the (almost ever) positive drift term we get a less conservative estimate of R-CVAR, but the picture changes:**
  - → The opposite is true for low level of active risk: now R-CDAR is substantially more conservative then R-CVAR
  - → Both R-CDAR and R-CVAR get bigger with higher level of active risk
  - → No stable relationship between the two measures.
- **It seems that realized R-CDAR is more conservative than R-CVAR with:**
  - Excess Kurtosis
  - Autocorrelation
  - Low levels of active risk

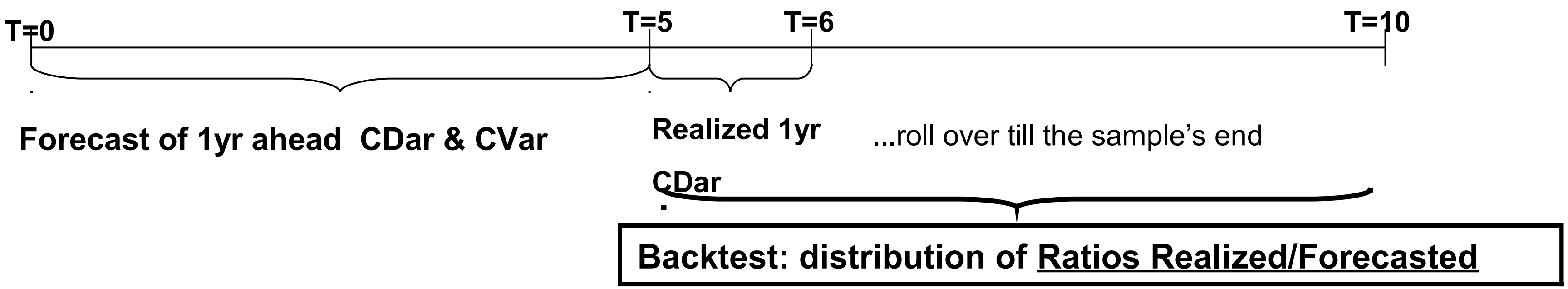


Quarterly Rebalancing (only if A.A. weights are over the specified threshold: +-5%)

Active Bond Weights	yearly excess performance	parametric var con drift	hist var con drift	ratio CDD/parametric CVar con drift	ratio CDD/hist CVar con drift	skewness	kurtosys	autocorr
-30,0%	-1,1%	-15,3%	-17,8%	88,5%	76,3%	0,09	1,67	0,02
-25,0%	-0,6%	-12,2%	-14,2%	84,6%	72,5%	0,04	1,40	0,03
-20,0%	-0,1%	-9,4%	-10,5%	88,9%	80,2%	0,15	1,49	0,01
-15,0%	0,4%	-6,5%	-7,1%	88,3%	81,2%	0,19	1,52	0,00
-10,0%	0,6%	-4,1%	-4,1%	78,0%	78,0%	0,25	1,54	0,00
-5,0%	1,0%	-1,5%	-1,5%	71,9%	71,2%	0,38	2,48	-0,01
-2,5%	1,2%	-0,5%	-0,5%	**111,7%**	**100,7%**	**0,48**	**4,02**	**0,00**
-1,0%	1,3%	-0,1%	-0,2%	**291,6%**	**172,6%**	**0,56**	**13,50**	**0,04**
0,0%	1,4%	-0,1%	-0,3%	**451,4%**	**145,0%**	**0,62**	**23,85**	**0,07**
1,0%	1,5%	-0,2%	-0,7%	**274,3%**	**83,5%**	**0,61**	**27,57**	**0,09**
2,5%	1,6%	-0,6%	-1,1%	**145,0%**	**75,0%**	**0,52**	**22,64**	**0,09**
5,0%	1,7%	-1,4%	-2,2%	**113,9%**	**73,5%**	**0,35**	**13,37**	**0,07**
10,0%	2,2%	-3,4%	-4,2%	**97,3%**	**78,8%**	**0,22**	**6,61**	**0,05**
15,0%	2,4%	-5,5%	-6,2%	90,8%	79,8%	0,12	4,44	0,04
20,0%	2,6%	-7,6%	-8,8%	87,5%	76,0%	0,07	3,49	0,04
25,0%	2,7%	-9,8%	-11,3%	85,7%	74,1%	0,04	2,97	0,03
30,0%	3,2%	-11,9%	-13,9%	84,3%	72,2%	-0,06	3,19	0,03

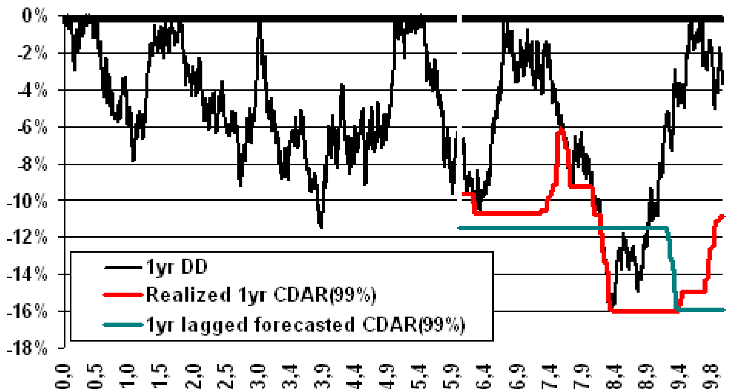
3.4 Backtesting R-CDAR

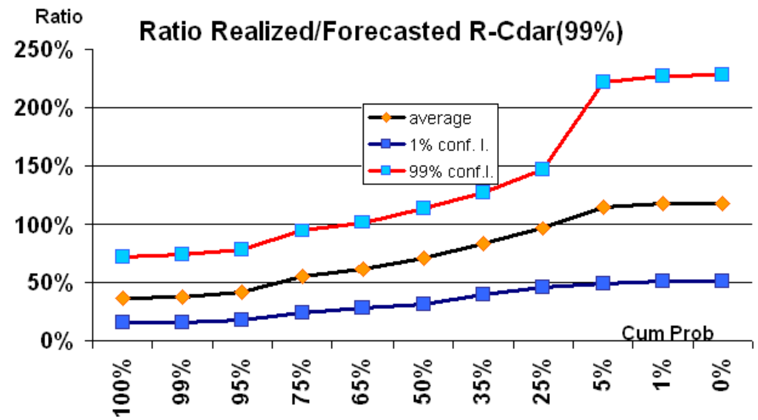
- **According to our record of backtest simulations the R-CDAR has to be handled with care.**
- **We test the following ratio:**
  - **Realized R-CDAR / Forecasted R-CDAR with**
    - 10yr of daily data (2500 days, i.e. 250 open mkt days x year)
    - 500 sim for each of the following distributions
    - "**good**" Informatio Ratio of **0,5**:   N(1%;2%); N(2%;4%); N(3%;6%)
    - "**bad**" Information Ratio of **-0,5**:   N(-1%;2%); N(-2%;4%); N(-3%;6%)
    - "**zero-sum game**" : N(0%;2%)
    - 1st 5yrs for parameters estimate; 2nd 5yrs to backtest out-of-sample the ratio realized/forecasted R-CDAR

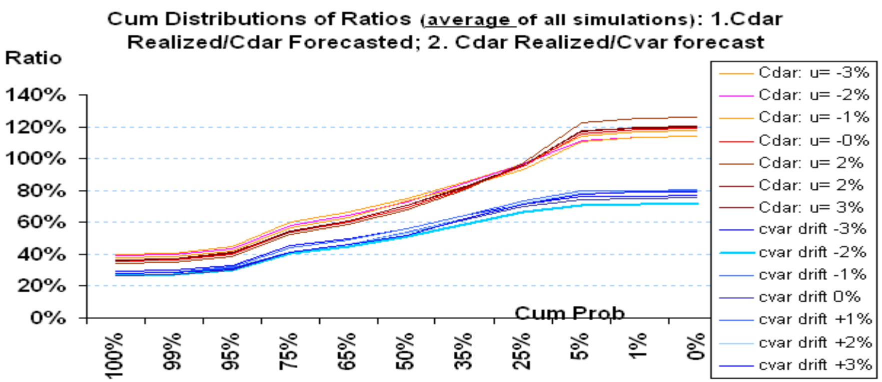


3.4 Backtesting R-CDAR

- **Our findings:**
  - → **Although on average the forecast makes a good job, the dispersion of Ratio is high**
  - → **It seems R-CVAR to be on average too conservative**







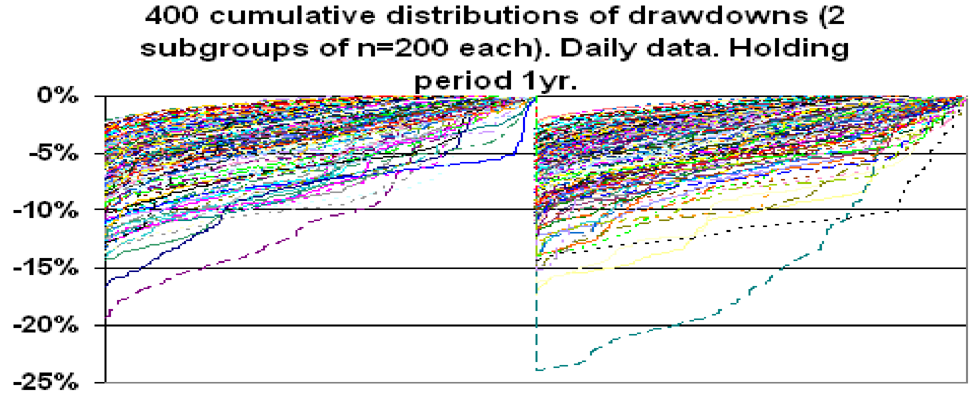
3.5 The boostrap R-CDAR

- **Our proposal would challenge the high uncertainty in the R-CDAR(99%) estimates**

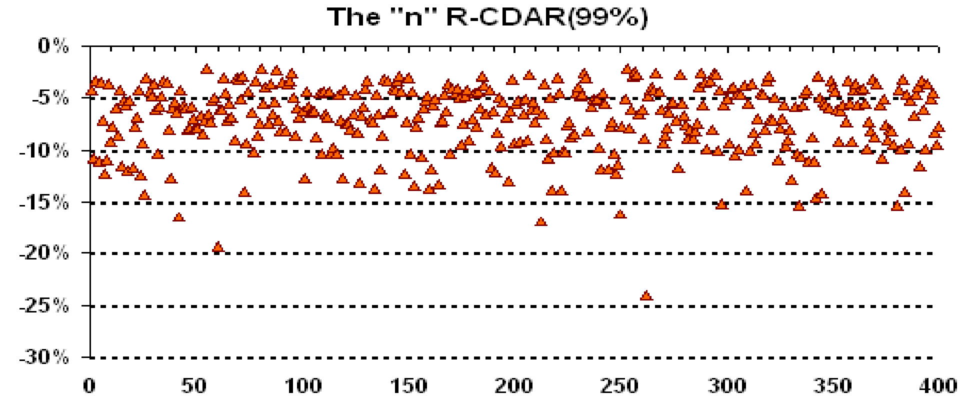
- **The Boostrap R-CDAR:**

  1. **Simulation of “n” (400) paths of cumulative excess return (block boostrapping on daily historical data)**
  2. **Calculate “n” distribution of relative drawdown**
  3. **Calculate the “n” R-CDAR(99%) one for each of the “n” distributions**
  4. **Establish a tolerance level by which extract, from all possible “n” R-CDAR(99%), the one that will be our best guess forecast of 1 year ahead R-CDAR (99%)**

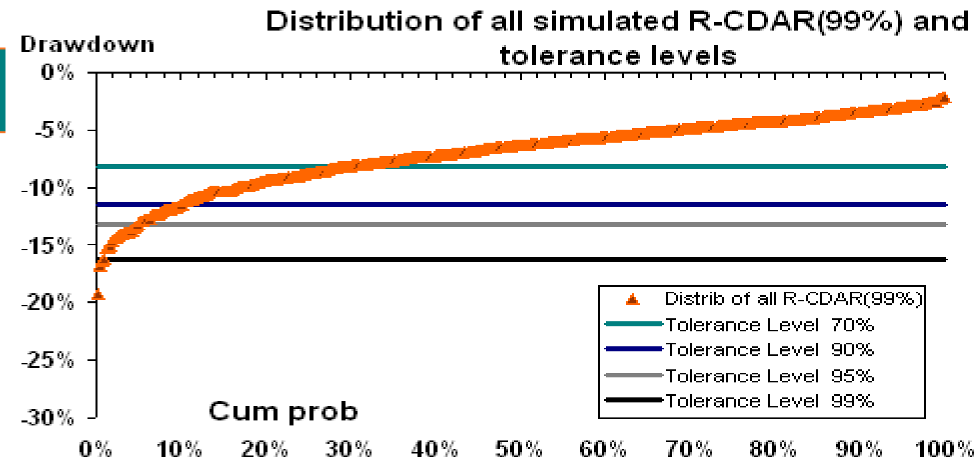
2



3



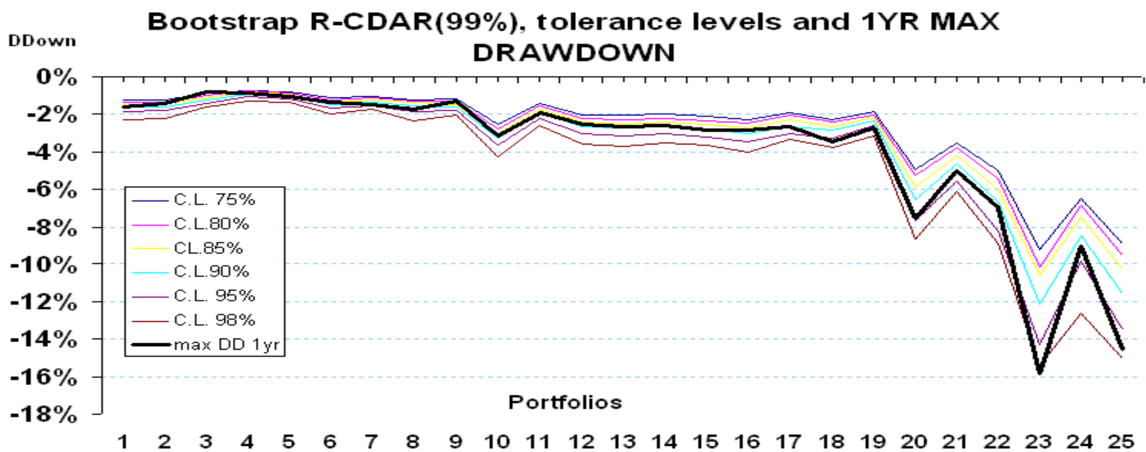
4



3.5 The boostrap R-CDAR

- **Here we try to investigate a proper in-sample tolerance level**
  - **In-sample test of 1yr bootstrap R-CDAR(99%):**
  - **25 portfolios (*): equity weight from 0% to 45%. 6yr of hist. daily data (1999-2004).**
  - **Tolerance levels on R-CDAR(99%) between 50% and 98%.**

  → We find that:
  - **for low level of max drawdown (<5% yearly) tolerance level of 85% offers a sufficient coverage;**
  - **for bigger drawdwons the tolerance level must be between 90% and 98%.**



3.5 The boostrap R-CDAR

- **Here we try to investigate a proper out-of-sample tolerance level**
  - Out-of-sample test of 1yr bootstrap R-CDAR(99%):
  - 3 assets: GBP/USD; 10yr Bund Future; S&P500.
  - Tolerance levels on R-CDAR(99%) between 50% and 99%.

  → Highlighted in yellow: the nearest upper tolerance level such that the realized 1yr ahead R-CDAR(99%) is equal or lower than the corresponding highlighted CDAR(99%) number

GBP/USD curncy

			Realized 1yr ahead CDAR(99%)		SIMULATED OUT-OF-SAMPLE FORECAST of 1yr ahead R-CDAR(99%) (Forecasted CDAR(99%) as a function of Tolerance Levels)							
Estimation period	Yearly Volatility in the estimation period	1yr CDAR(99%) in the estimation period	Year	Realized CDAR(99%)	<=TL 50%	<=TL 75%	<=TL 80%	<=TL85%	<=TL 90%	<=TL 95%	<=TL 98%	<=TL 99%
1/1981 - 12/1985	11,96%	-26,95%	1986	-9,80%	**-16,97%**	-20,70%	-21,44%	-22,75%	-24,54%	-26,72%	-29,27%	-30,25%
1/1982- 12/1986	11,32%	-26,84%	1987	-7,17%	**-14,2%**	-17,6%	-18,3%	-19,5%	-21,7%	-24,3%	-26,5%	-28,2%
1/1983 - 12/1987	11,38%	-26,84%	1988	-12,58%	-11,5%	**-14,2%**	-15,0%	-16,4%	-18,3%	-20,0%	-21,4%	-24,0%
1/1984 - 12/1988	11,71%	-26,84%	1989	-16,63%	-10,9%	-13,4%	-14,1%	-15,1%	**-16,9%**	-19,1%	-21,0%	-22,0%
1/1985 - 12/1989	12,14%	-17,61%	1990	-6,71%	**-10,6%**	-12,9%	-13,9%	-15,0%	-16,5%	-18,2%	-21,6%	-23,9%
1/1986 - 12/1990	10,52%	-17,61%	1991	-19,97%	-9,8%	-11,9%	-13,0%	-14,1%	-15,3%	-18,2%	**-23,7%**	-25,9%
1/1987 - 12/1991	10,96%	-19,33%	1992	-24,97%	-10,8%	-13,6%	-14,5%	-15,5%	-16,8%	-18,8%	-22,9%	**-25,9%**
1/1988 - 12/1992	11,96%	-24,14%	1993	-8,72%	**-14,6%**	-18,6%	-20,3%	-22,2%	-25,3%	-29,7%	-32,2%	-34,8%
1/1989 - 12/1993	12,39%	-28,75%	1994	-5,71%	**-15,1%**	-19,6%	-21,4%	-23,1%	-24,9%	-28,7%	-34,7%	-37,1%
1/1990 - 12/1994	11,59%	-28,75%	1995	-6,83%	**-13,8%**	-18,1%	-18,9%	-20,9%	-22,7%	-26,8%	-30,8%	-31,7%
1/1991 - 12/1995	11,38%	-28,75%	1996	-3,68%	**-13,7%**	-17,7%	-19,3%	-20,9%	-23,4%	-27,8%	-32,1%	-33,6%
1/1992 - 12/1996	10,33%	-28,75%	1997	-7,84%	**-11,4%**	-15,3%	-16,3%	-17,4%	-20,4%	-23,9%	-28,6%	-32,3%
1/1993 - 12/1997	8,87%	-8,40%	1998	-4,47%	**-8,6%**	-11,0%	-12,1%	-13,0%	-14,3%	-16,6%	-20,7%	-22,9%
1/1994 - 12/1998	7,57%	-8,00%	1999	-6,52%	**-6,8%**	-8,3%	-8,8%	-9,5%	-10,5%	-12,0%	-14,3%	-16,0%
1/1995 - 12/1999	7,52%	-9,34%	2000	-15,49%	-7,1%	-8,7%	-9,4%	-10,2%	-11,3%	-12,7%	-14,4%	**-16,1%**
1/1996 - 12/2000	7,50%	-15,67%	2001	-8,49%	-8,1%	**-10,1%**	-10,8%	-11,7%	-13,0%	-14,9%	-17,3%	-18,3%
1/1997 - 12/2001	7,78%	-15,67%	2002	-3,69%	**-9,7%**	-11,8%	-13,1%	-14,3%	-16,2%	-19,0%	-21,7%	-23,8%
1/1998 - 12/2002	7,49%	-15,67%	2003	-6,69%	**-8,5%**	-10,3%	-11,3%	-12,3%	-13,8%	-18,1%	-24,5%	-26,3%
1/1999 - 12/2003	7,63%	-15,67%	2004	-7,74%	-7,7%	**-9,9%**	-10,6%	-11,2%	-12,1%	-13,6%	-17,1%	-20,1%

3.5 The boostrap R-CDAR

→ **Highlighted in yellow: the nearest upper tolerance level such that the realized 1yr ahead R-CDAR(99%) is equal or lower than the corresponding highlighted CDAR(99%) number**

10yr Bund future

Estimation period	Yearly Volatility in the estimation period	1yr CDAR(99%) in the estimation period	Realized 1yr ahead CDAR(99%): Year	Realized CDAR(99%)	SIMULATED OUT-OF-SAMPLE FORECAST of 1yr ahead R-CDAR(99%) (Forecasted CDAR(99%) as a function of Tolerance Levels): <=TL 50%	<=TL 75%	<=TL 80%	<=TL85%	<=TL 90%	<=TL 95%	<=TL 98%	<=TL 99%
1/1991 - 12/1995	5,31%	-12,75%	1996	-6,52%	-4,4%	-5,3%	-5,7%	-6,1%	-6,9%	-8,7%	-11,0%	-12,1%
1/1992 - 12/1996	5,56%	-12,75%	1997	-4,25%	-5,1%	-6,5%	-6,8%	-7,6%	-8,6%	-9,5%	-11,4%	-12,4%
1/1993 - 12/1997	5,73%	-12,75%	1998	-3,72%	-5,4%	-6,8%	-7,3%	-7,8%	-9,0%	-10,3%	-12,4%	-13,6%
1/1994 - 12/1998	5,87%	-12,75%	1999	-12,28%	-5,4%	-6,7%	-7,2%	-7,8%	-8,5%	-10,0%	-11,7%	-12,3%
1/1995 - 12/1999	5,58%	-11,82%	2000	-2,62%	-5,6%	-6,5%	-7,3%	-8,0%	-9,8%	-13,3%	-16,3%	-19,3%
1/1996 - 12/2000	5,42%	-12,54%	2001	-5,31%	-5,3%	-6,6%	-7,1%	-7,6%	-8,4%	-10,2%	-11,5%	-12,6%
1/1997 - 12/2001	5,25%	-12,54%	2002	-3,91%	-5,4%	-6,9%	-7,3%	-7,8%	-8,6%	-9,9%	-11,6%	-12,9%
1/1998 - 12/2002	5,29%	-12,54%	2003	-6,88%	-6,0%	-7,4%	-8,0%	-8,7%	-9,7%	-11,4%	-18,6%	-19,7%
1/1999 - 12/2003	5,55%	-12,54%	2004	-4,02%	-6,4%	-8,0%	-8,4%	-9,4%	-10,4%	-11,7%	-14,6%	-17,0%

S&P500

Estimation period	Yearly Volatility in the estimation period	CDAR(99%) in the estimation period	Realized 1yr ahead CDAR(99%): Year	Realized CDAR(99%)	SIMULATED OUT-OF-SAMPLE FORECAST of 1yr ahead R-CDAR(99%) (Forecasted CDAR(99%) as a function of Tolerance Levels): <=TL 50%	<=TL 75%	<=TL 80%	<=TL85%	<=TL 90%	<=TL 95%	<=TL 98%	<=TL 99%
1/1981 - 12/1985	13,70%	-20,21%	1986	-9,2%	-11,96%	-14,69%	-15,41%	-17,64%	-19,65%	-22,42%	-24,56%	-26,85%
1/1982- 12/1986	13,91%	-14,82%	1987	-33,3%	-10,2%	-12,2%	-13,3%	-14,3%	-15,0%	-17,1%	-20,0%	-21,0%
1/1983 - 12/1987	18,07%	-31,37%	1988	-7,4%	-14,1%	-17,5%	-22,2%	-26,7%	-30,2%	-33,2%	-41,2%	-42,5%
1/1984 - 12/1988	18,71%	-31,37%	1989	-7,3%	-14,0%	-16,3%	-19,6%	-26,4%	-29,8%	-33,1%	-39,0%	-43,8%
1/1985 - 12/1989	18,76%	-31,37%	1990	-19,4%	-13,7%	-15,9%	-19,8%	-26,3%	-29,8%	-32,3%	-37,5%	-46,4%
1/1986 - 12/1990	19,53%	-31,37%	1991	-5,6%	-17,3%	-23,4%	-26,7%	-29,8%	-34,1%	-39,7%	-45,4%	-49,6%
1/1987 - 12/1991	19,45%	-31,37%	1992	-5,7%	-14,9%	-18,2%	-21,4%	-27,5%	-30,6%	-34,3%	-39,3%	-45,1%
1/1988 - 12/1992	14,05%	-18,08%	1993	-4,5%	-10,3%	-12,5%	-13,6%	-14,5%	-15,8%	-18,6%	-21,6%	-23,4%
1/1989 - 12/1993	12,45%	-18,08%	1994	-8,6%	-9,4%	-11,3%	-12,1%	-12,9%	-14,8%	-17,8%	-20,8%	-24,4%
1/1990 - 12/1994	11,85%	-18,08%	1995	-2,4%	-9,8%	-12,3%	-13,1%	-14,6%	-15,6%	-18,2%	-21,1%	-24,2%
1/1991 - 12/1995	10,16%	-8,00%	1996	-7,6%	-6,2%	-7,4%	-7,8%	-8,5%	-9,2%	-10,7%	-12,8%	-13,5%
1/1992 - 12/1996	9,48%	-8,01%	1997	-9,9%	-6,4%	-7,6%	-8,2%	-9,1%	-9,8%	-11,0%	-13,3%	-15,1%
1/1993 - 12/1997	11,62%	-8,44%	1998	-19,0%	-7,0%	-8,5%	-9,0%	-9,6%	-11,2%	-11,9%	-15,1%	-15,6%
1/1994 - 12/1998	14,17%	-17,34%	1999	-11,7%	-9,5%	-11,9%	-12,8%	-13,4%	-14,5%	-16,4%	-20,4%	-23,3%
1/1995 - 12/1999	15,66%	-17,34%	2000	-16,7%	-11,8%	-12,7%	-13,6%	-15,0%	-16,7%	-21,7%	-25,7%	-28,2%
1/1996 - 12/2000	18,13%	-17,34%	2001	-28,5%	-12,4%	-14,6%	-15,6%	-17,0%	-18,9%	-21,6%	-24,8%	-30,1%
1/1997 - 12/2001	19,77%	-28,64%	2002	-33,1%	-15,6%	-18,4%	-19,7%	-21,2%	-24,1%	-29,3%	-32,7%	-35,7%
1/1998 - 12/2002	21,40%	-31,62%	2003	-13,7%	-19,9%	-24,7%	-26,7%	-28,0%	-30,1%	-34,3%	-38,4%	-40,7%
1/1999 - 12/2003	20,85%	-31,63%	2004	-8,1%	-19,9%	-24,7%	-26,5%	-28,9%	-31,0%	-34,3%	-37,0%	-42,2%

3.6 Conclusions

- **PROS of R-CDAR:**
  - No more gap between the 3 components of risk analysis
  - Very natural risk measure from an investor's standpoint
  - It is written on the regulatory novel of the high water-mark
  - It doesn't need distributional assumptions, nor drifts and correlation estimations
  - No sub-additivity problem; no needs of scaling law assumptions (ie sqrt(t))
  - Easier risk monitoring and management in presence of threshold agreements

- **CONS of R-CDAR:**
  - Computationally intensive
  - No closed-form solution for portfolio risk budgeting/contribution

- **Further researches:**
  - Drawdowns and time diversification vs level of active risk
  - Balancing responsiveness and stability of the model
  - Drawdown estimation with not-normal distributions